The whetstone of witte,
which is the seconde parte of
Arithmetike: containing the traction
of rootes: The cosike practise,
with the rule of Equation: and
the 2002kes of Surde
Numbers.

Though many stones do beare greate price,
The whetstone is for exercise
As needesfull, and in woorke as strange:
Dulle thinges and where it will so change,
And make them sharpe, to right good use:
All artefynes knowe, they cannot abuse.
But use his helpe yet as men see,
Now sharpenesse someth in it to bee.
The grounde of artes did brede this stone:
His use is greate, and more then one.
Here if you lift your wittes to whette,
Moke sharpenesse therby shal you gette.
Dulle wittes hereby doe greatly mende,
Sharpe wittes are finisht to their full end.
Now praise, and proue, as you doe finde,
And to your self be not vnkinde.

These Bookes are to bee solde, at
the Weste doore of Poules,
by Iohn Kyngstone.
To the right worshipful, the governors, Consulles, and the rest of the compa- 
piane of venturers into Moscouia, Robert Ke-
cade Phisitian, wissheth healthe with 
continuall increase of comodidi 
tie, by their woorthe and 
famous trauell.

Wil not, nether ought I to euilly to judge of my countrie, that learning 
here can have no libertie: but by aide of frendshippe, oz strengthe of po-
wer. Foz as Englaunde did neuer wante learned 

witthe, so at this tyme I doubt not, but there 
be a great multitude, that delstitially embrace 
all kinde of knowledge, and frendely are af-
fected toward the furthere of it. And ther-
foxe I dare sate, thei can not malicke me, whi-
che am so willyng to helpe the ignoraunte, ac-
cordyng to my gifte and simple talente, whe-
by also this moche praise I maie iustely crave, 
to haue the commendation and rewarde of a 
solliciter in this cause. Foz though my trauell 
can not moche profite them, that be well lear-
ed, yet doeth it excite the beste learned, to re-
member their duttie to their countrie: and to 
be a shamed, that thei hauynge so greate habi-
litie, shall be founde more slacke to aide their 

countrie, then he that hath smaller knowledge, 
a, ii, and
The Epistle

...and lesse occasion otherwaies. Accoodyng as men have receiued, so are the bounde to payd. These excellent gifts are not lent unto me, to be hidden. And there are a great multitude that thirst, and long moche for soche aid? For bothe these causes I saie, that naturall bode to our countrie doeth chalenge it: and for that the honeste desiers of so many good natures so moche requireth it. I exhort them that be bestable, to take from me this chargeable woork, and to further their countrie men, as equitie would. And in the meane season while I see them so slacke, let them not bee offended with me, for preventing them. For better it is that a simple Cooke doe prepare thy breakefast, then that thou shouldest goe a hungered to bedde. Yea better it is to have some grosse repaste, then to serue for hunger. And the common Cooke will finde smalle faulte of wante, as long as they see any man serue their expectation. So that for this cause also, that my painses for a time, doeth excuse other finer wittes, thei ought to render me some thankes again. But if thei state for feare of tauntes, and backynge of cures, their courage is smalle. If thei misdoubte the grateful acceptance of their studies, thei doe injurie to their countrie. For whoe can doubt but so ciuile a countrie, will thankfully receive, and moste freely recompense the trussile, of soche as studie for their benifite,
Dedicaturie

benefite, and serveth their necessarie commodities. This persuasion maketh me so bold, that I can not thinke it needesfull, to seke any protector, for this or any like woork. Sith every good man will offer hymself, to defende that, whereby his native countrie is benifited. Excepte at some tyme, by excitation of the furries, some naughtie natures doe practice their fraude, to bereste the reallne of some singu-lare commoditie. But as I feare no soche, so at this tyme I seke no soche aide against the. Yet for rethrowing of friendship, and grate-full remembrance, I would doe noe lesse, but sende this Booke to soche as I thought, not onely to deserue it, but also would gladly receiue it. And if it may perceiue, that you doe accepte it ( as I doubt not ) with as good a will, as I doe sende it, I will for your pleasure, to your countre, and for your commoditie, shortly set for the soche a booke of Nau-gation, as I dare saie, shal partlie satifie and contente, not onely your expectation, but also the desire of a greate number beside. Where- in I will not forgett specially to touche, bothe the olde attemptes for the Northlie Navigations, and the later good adventure, with the forturnate success in discoverynge that voyrage, which use men before you durste attempte, sith the tyme of Kyng Alured his reigne. I meane by the space of, 700 yerke, Nothere ever a.m. any
The Epistle

any before that tyme, had passed that voyage, except one only Othhere, that dwelte in Hal- gollandes whose reported that ioyney to the no- ble Kyng Alurede: As it doeth yet remaine in aunciente recorde of the olde Saxon tongue. So that if you continue with corage, as you have well begun, you shall not onely winne greate riches to your selues, and bryng wonderedfull commodities to your coutrie. But you shall purchase there with immortall fame, and be praised for ever, as reason would: for opening that passage, that shall profite to many. In that Boke also I will shewe certain means, how without greate diffcultie, you maie saile to the North West Indies. And so to Camul, Chinchita, and Baloz, whiche bee coutries of greate commodities. As for Chatai lieth so farre within the lande, toward the South Indian seas, that the ioyrne is not to be attempted, untill you be better acquainted with these countries, that you must first arrive at. But these thynges come in this place untimely. I praye you accepte frendely in the meane reason this Booke, whiche will bee a greate aide to the well understandynge of the reste that is behinde. And as I shall understande your desire, so will I haste the other. God prospere well your endeouer, and sende you soche good successe, as so worthie adven- ture doeth deserve: Whiche I doubt not will in sue,
Dedicatorie.

insue, if cankered malice of some spitefull stomackes doe not prevaile, as thei can not cease to practice, to hinder your commoditie, and deface your travel. But as it is ever seen, and therefore commonly known, that enuie doeth still repine at glorie, so ought all honeste hartes, to prosecute their good attemptes, and contempne the ballynge of dogged curreis. So fare you well. And loue hymagain, that delighteth and studieth to farther your commoditie.

At London the xii. daie of November, 1557.
Although number be infinite in increasing; so that there is not in all the world, any thing that can exceed the quantity of it: Neither the grave on the ground, nor the drops of water in the sea, no not the small grains of sand through the whole mass of the earth: yet make it seem by good reason, that no man is so expert in Arithmetike, that can number the commodities of it. Wherefore I make truly safe, that if any imperfection bee in number, it is because that number, can scarce number, the commodities of itself. For the more that any expert man, doth weigh in his mind the benefits of it, the more of them shall he see to remain behind. And so shall he well perceive, that as number is infinite, so are the commodities of it as infinite. And if any thing doe make exceed the whole world, it is number, which so farre surmounteth the measure of the world, that if there were infinite worlds, it would at the full comprehend them all. This number also hath other prerogatives, above all naturallly thynges, for neither is there certainty in any thyng without it, nor ther good agreeemente where it wanteth. Wherefore no man can doubt, that hath been accustomed in the booke of Plato, Aristotle, and other aunciente Philosophers, where he shall see, how they seach all secrete knowledge and hid mysteries, by the side of number. For not onely the constitution of the whole world, doth thei referre to number, but also the composition of b.i. man,
manie, yea and the beste substance of the soule. Of which thei profess to knowe no more, then theye by the benifice of number attaine. Furthmore, for knowledge and certaintie in any other thyng, that manners witte can reche unto, there is no possibilitie without number. It is confessed amonge the all men, that knowe what learning now athen, that besides the Mathematicall artes, there is no infallible knowledge,excepte it be browen of them, And amonge them, it is sufficiently known, and well declared by Nicomachus, and diverse other writers, that Arithmesike is the fountaine of all the other, and their ground and bonde, as he calleth it. If any man will lye, that Divinitie, Lawe, and Physike, may bee had without it: 02 that the take little aide therby. Although I have before this time answered thereto, yet now I say again: that in Divinitie there are greate hidde secretes in numbers. So that diverse excellent Divine, have written whole Bookes of the mysteries of numbers. And some of their Bookes intituled: The Divinitie of Numbers. But what Christen manne is ignorant, that betweene Trinitie and unitie, doeth consiste the fulle grounde of al Divinitie: Wherefore I neede not to allege the other hollie and sacred Pombers. Save that, 7. will not permitt me to passe it with silence. In whiche is contained, not onely the secretes of the creation of all thynges: and the consummation of the whole worlde againe, with the state of eternitie: But also by it is the Sabbathe rest, and therby the full life and conversation of godlie persones, represented and instinuate. In Lawe twoe kyndes of Justice are the somme of the studie: Justice Distributive, and Justice Commutative, which termes I doe, as beste knowne in that arte: But what is any of the bothe without Pomber? I have laid in an other place (as I learned of that noble Philosopher Aristotell) that
that the knowledge and distinction of Geometrical
and Arithmetical proportion be not well observed,
there can no justice well be executed. And how of-
ten the ministers of the Lawe use aide of Physike. I
neede not repete, because none but madde men doubt
of it. And as for Physike, without knowledge and
aide of nomber is no thyngye. Wee see that nature in
generation, beth of manne and beastes, yea and of all
thynges els dooth observe nomber cruely. As well in
the tyme of fo-znation, as in the monethe of quicken-
nyng, and of birth. The mysteries of the seuenth and
ninethe monethes are sufficient testimonies therein.
Beside that from the souther monethe til the seuenth
many thynges be permitted, that els be not conven-
tiente. For the bse of the pulse, and for criticalle da-
yes, beside the proportion in degrees in simple medi-
cines, and mixture of compounde medicines, and oth-
er infinite matters, what nomber can doe and what
aide it gluth, onely the ignoraunte doe doubte.

But where can there bee any better testimonie for
Nomber, then that the celestialle bodies doe kepe an
unfallible nomber, in all their wonderfulle motions:
By meanes wherof, mannes witte is hable to at-
taine the knowledge of them. As by the Arithmetical
tables, of their motions it is easily knowne. There-
fore and for that we see the yere, and all the distincti-
on of times, beside the common bse of traske betweene
manne, to depende of nomber, wee musste neade not
onely confess it to bee, as it were the onely state of
all natures woorkes, and of all ciuitie: but wee must
also honour and reverence it, as often as wee duly
remember the excellencie and benifique of it. Was not
Nomber, thinke you, wonderfully honoured, when
noe name was thought moare meter for God, then
the name of Nomber; I meane. 1. and. 3. the name of
the Trinite. But to come to moare familiare ma-
ners,
THE PREFACE.

...ters, I will sие, as Plato saith in his Booke De Summo bono. Take away Arithmetike, with measure and weights, from all other artes, and the reste that remaineth is but base, and of noe estimation. Wherealthough Plato doe name three things in apperance, that is Number, Measure, and Weighte. What are Measure and Weighte, but number applied to seuerall uses: For Measure is but the numberyng of the partes of lengthe, bredthe, or deepthe. And so weighte (as here it is taken) is the numberyng of the heuitell of any thyng. So that if number were withdrawen, no manne could either measure, or weigh any quantitie. And therefore it must followe: that number onely maketh all artes perfecte, and worthie estimation: sayng that without it, all artes are but base, and without commendation. This may suffice for the litle commendation of Arithmetike. But yet one commoditie more, whiche all menne that studie that arte, doe se, I can not omitte. That is the fyllyng, sharpenying, and quickening of the witte, that by practice of Arithmetike doeth infuie. It teacheth menne and accusometh them, so certainly to remember thynges paste: So circumspectly to consider thynges presente: And so prouidently to so; see thynges that followe: that it may trucly be called the Eile of witte. Ye it may aptly be named the Scholehouse of reason. The like judgemente had Plato of it, as appeareth by his woordes in the seuenth booke Dene publica. Where he saith thus: Thei that be apte of nature to Arithmetike, bee readie and quicke to attaine all kindes of learning. And thei that bee dulle witted, and yet bee instructed and exerced in it, though thei gette nothing els, yet this shall thei all obtain, that thei shall bee more sharper witte, then thei were before. What a benifite that onely thyng is, to have the witte whetten and sharpened, I neare not travell to declare, Sith all menne confesse it to be as greate as male be. Excepte any witless person...
some thinke he maie bee to wise. But he that moste
scareth that, is leaste in danger of it. Wherefore to
conclude, I see moare menne to acknowledge the ben-
fitte of nonber, then I can espie willyng to studie,
to attaine the benfittes of it. Manye praiseth, but fevere
doone greatlye practiseth: onlesse it bee for the bulgare
practise, concerning Merchanddes trade. Wherein
the desire and hope of gaine, maketh many willyng to
sustaine some travell. For aide of whom, I did sette
forth the fiste parte of Arithmetike. But if thei knewe
how farre this seconde parte, dooth excelle the fiste
parte, thei would not accompliseth any tyme loste, that
were imploied in it. Pea thei would not thinke any
byme well bestowed, till thei had gotten soche habili-
tie by it, that it might be their aide in al other studies.
And if Plato doe require Arithmetike, as a specialle and
a necessarie qualitative in hym, whom he would admitte
as a cizeen in his politike tounck: How maie wee
thinke of our selues, that desire to gouerne other, and
yet can scanty skille of common nonber? So farre are
many, pea moste parte of vs from cunninge in non-
ber. Plato thinketh noe manne hable to bee a good ca-
pitaine, excepte he bee skilfull in this arte: And wee
accompliseth noe parte of those qualitative, that bee re-
quired in any soche manne. Howbeit for the better
triall thereof, I have in this Booke framed some of
the questions in soche sorte, as thei maie approue the
bye of this arte, not onely good for capitaines, but al-
so moste necessarie for theim. So that without it, thei
can not Marshall their battaile, neither be we their en-
nemies campe or sorte. And if I shall sake as I thinke,
without it a capitaine is noe capitaine. In this booke
what I have written, for the aide of all menne, and
namely soche of my countrie menne, that understand
nothynge but English, I neade not to repete particular-
larely, but remitte them to the booke it self, to see it at
by. large
large. Only this male I sate: that as I have done in other arses, so in this I am the first venturer, in these darke maters. Wherefore I trust the that be learned, and happen to reade this worke, wil beare the moare with me, if the finde any thyng that the doe mistake: Wherein if the will ble this curtelie, either by wise tyngge to admonishe me thereof, either theim selves to sette so the a moare perfecte worke, I will thynke them praise worthie. But if any manne will be so haftie, other to blame that, Whiche he is not hable to amende, or to condemne that, Whiche he did neuer understande: As some ofte tymes doe of a sonde curtelie, I will wishe hym a better witte, and moare modeulie. And to prevent all soche seueres Judges, I thought it good to admonishe you befoze, that by occasion of trouble upon trouble, I was hindered from accomplishing this worke, as I did intende. But yet is here moare, then any manne might well looke for at my hande, if the did knowe and consider myne estate. And this moche moare I sate: that if I male perceiue, that this Booke bee as well receiued, as the firste parte was, I will strive moche, to steal from my troubles to moche tyme, as to set out the reste of this arte, moare completely in English, then euer I sawe it in any tongue, hether to doen: trust thereto assuredly. And wishe hym good, that trauelth so thy benifite.
Of the use of Cofe.

One thyng is nothyng, the proverbe is, whiche in some cases doeth not misse.
Yet here by woorkyng with one thyng, Soche knowledge doeth from one roote spryng,
That one thyng maie with right good skille, Compare with all thyng: And you wille
The practice learn; you shal sone see,
What thynges by one thyng knownen maie bee.

To the curious scanner.

If you ought finde, as some men maie, That you can mende, I shall you praie,
To take some paine: so grace maie sende, This worke to growe to perfecte ende.
But if you mende not that you blame, I winne the praise, and you the shame.
Therefore be wise, and learne before, Sith slander hurteth it self moste sore.
See your desire can not
bee satisfied, neither your re-
quest stated, untill I maie lu-
uely answer you, that I can
teache you no more: whiche
answer were maie stale your
quest, although it content not
your desire.

Scholar. I beseech God of
his mercie, to withstande all suche occasion: except it
maie be more to your owne contention and profite,
then it would be pleaasunt to the louers of learning.

Mater. Yet a just excuse maie stande so, my de-
claration: As if ignozaunce doe enforce me to stale
my trauell.

Scholar. Your owne ignozaunce, I trust, you will
not allege: and as so, the ignozaunce of other, it ought
to bee no stale; sith the ignozaunte multitude doeth,
but as it was ever wonte, enue that knowledge,
whiche they can not attaine, and wilde all men igno-
raunt, like unto themself, but all gentle natures,con-
tempteth suche malice: and despiseth theim as blinde
wozmes, whom nature doeth plague, to stay the poi-
sone of their venemous styenge.

Mater. We shall not neede to stande on this talke,
but trauell with knowledge to vanquishe ignozaunce:
And beleue that the pricke of knowledge, is more of
force then the styenge of ignozaunce: yea, the pointe in
A. 1. Geometric,
The seconde parte

Geometric, and the unitie in Arithmetike, though bothe be indivisible, doe make greater woorkes, yincrease greater multitudes, then the holye bande of ignorance is hable to withstand.

Scholar. Our talke groweth well to our mater. I beseke you thersoone, with that unitie beginne, and bulde on it your woorkes, as a foarte against ignorance. Master. Unitie is of it selfe indivisible, and yet is it in all partes of the worlde, and in every thing. Pea, the worlde it selfe consisteth of unitie, is named of unitie, was made by unitie, and is preserved by unitie, and onely ignorance with her broode secluded from unitie, so that of it to repeate the fulle foarte, would occupy muche time, and make great volumes.

Scholar. Sith unitie is so mightie, and of suche foarte (as you saie) what matie be thought then of number, which containeth a multitude of unities? And is nothyng els but a collection of unities.

Master. Unitie is the fountaine and originall of number, pea unitie by addition onely shall make a greater number, then any numbers can doe by multiplication. But this is maruellous, that no number repineth against division, till it come to an unitie: and then will it permit no farther division. And therefor it is said, that unitie dooth neither multiply nor divide.

And as al numbers maie be more or lesse, so the lesse is cuuer a parte or partes of the greater.

As 5 unto 10 is a parte named a halfe: but unto 7, 5 is not a parte, our partes, and is called 1. So 8 to 24 is a parte that is 1; but unto 36 it is partes, that is 1. 3.

Scholar. I perceave, you call it a parte, when the numeratoz in the fraction (reduced to the smallest) is an unitie. And when the numeratoz is a number, then that fraction betokeneth partes of a number.

But I praise you, what varieties of numbers bee there principally to be considered in this arte?

Master,
of Arithmetike.

Master. Number is divided into diverse kindes. The firste division are whole numbers, and the onely of Euclide, vision of non Soetius, and other good writers are called numbers. Other are broken numbers, and are commonly called fractions. Of these bothe I have written in the firste and seconde partes of Arithmetike: So that I mighte seeme to curiose, to repeate any parte of it again.

But now in eche kinde of these, there are certaine The seconde numbers named Abstraste: and other called numbers division of numbers.

Abstraste numbers are those, whiche have no deno: Abstraste. mination annexed unto them. And those that have any denomination lowned to theim, are called Contraste Contraste. numbers.

Scholar. This I see to be a reasonable distinctio, and agreable to the signification of the names.

For as that number is contraste, from his generall libertie of signification, whiche is boide to one denomination, as in sayng. 10. grotes (where. 10. is reduced from the libertie of valowyng any other thing but grotes) so if it had no denomination addicted, it might then signifie the number of daces, or of miles, or any like thyng, as well as of grotes. For when I saye. 10: and doe not limitte any denomination, then is that. 10. abstracte and severed from all specialties, and standeth free to any name of thing.

But this (me thinketh) under your correction) can not extend to broken numbers: Which are called numbers with them their denomination: sayng the consiste of a numerator and a denominator.

Master. You seeme to fate well. And the like judgement doeth appere to be in some writers of this arte. But yet sayng that fractions maie have all other ar tificiall denominations, that whole numbers maie receive; and maie also bee without them; therefore must we either make a more curious distinction of that
The seconde parte

that name of denomination: or els wee must conclude fractions, sze the necessity of that name: or els thirdly, to noted contention, call them numbers contrate improperly.

Scholar. I attente thereto as reason would.

why fractions be not called Euclide, and the other learned men, refuse to accompte fractions emongest numbers.

Master. Because all numbers doe consiste of a multitude of unities: and every proper fraction is less then an unitie, and therefore can not fractions exactly be called numbers: but male be called rather fractions of numbers.

Scholar. In deede now that I doe waie the mater more exactly, it appereth that a fraction is not properly a number, but a connexion and conference of numbers, declaryng the partes of an unitie. For the numerator dooth signifie one nober, and the denominato two other: the denominator declaryng into how many partes the unitie is divided, and the numerato signifying that of those partes, not all, but so many only are to be takyn, as the numerator importeth.

Master. Well, then to procede, numbers abstracte are considered in three principal varieties: That is, first without comparison to any other number or figure. And that number male well be called number absolute.

Secondarily, some numbers bee used onely in relation to other, and therefor ought to bee called numbers relative.

Thirdly, many numbers are referred to some figure, that male rise by multiplication of their partes together, and that diversely. And those numbers therefor male bee called figuralle numbers.

Scholar. If I conceive your wordes rightly, this is your meaning: that when I saie, 10, 25, 100, or 200, &c. these numbers stand absolute from all denomination.
of Arithmetike.

minacion, and clere from all relation and comparison.

But when I saie, 6. is halfe of 12. and 15. is triple to 5. here the numbers beeyng compared together, are aptly called numbers relative: So if I saie, that, 16. is a square number, because it is made of 4. multiplied by 4. then is 16. here to be called a figuralle number.

Master. You take it well. Therefore will I briefly touche the members of every kinde.

First of absolute nombres, some are even nombres, and some are odd.

Scholar. All men knowe that. And farther, that Numbers, even numbers are those, whiche may be divided into 2 even, and odd qualities halves: and so can not odd numbers, without a fraction.

Master. Of this plaine easie thing, mark what followeth: a greater doubte dissolved. For if an odd number (as 7. for example) can not bee parted into 2. equalle numbers, eche beeyng halfe of 7. then; 7 1/2. which is commonly called the halfe of 7. is no number.

Scholar. It can not be denied. And so (I see now) no fraction can bee a number. This greater doubt is plainly dissolved, by a very certaine and moste knowne principle.

Master. Now farther. Of bothe these kindes of Numbers are numbers, some be compound, and some be simple and pounde, and uncompound. Compounde nombres are made by multiplication of 2. nombres together, and not by addition, though the name might seem to serve to bothe.

Scholar. So I perceive, that 5. is no compounde number, although it bee made by addition of 2. and 3. but 6. which is made by multiplication of 2. and 3.

Likewaies. 9. is compounde, because that 3. multiplied by 3. doth make 9.

And, 15. also is compounde by multiplying 5. and 3. together.

And hereby I se that 1. is not to be called a number number.

A.iii. 402
The seconde parte

For then all numbers above it, must needs be compound, because they consist all of unities.

Master. But yet by multiplication of 1, no other number is compound.

Scholar. By those words I am taught to knowe more, and speake better.

Master. Even numbers are yet diversely to be considered in their divisions, for although the great multitude of even numbers bee compound, yet 2 is composed truly an even number, originall, and uncompound.

So that it maye make other numbers, 8 is made of no numbers, but of unities onely, as all odd numbers are.

All other even numbers are compound, and are diversely divided, for some are even numbers evenly, and some are even numbers oddly, and some are even numbers both evenly and oddly. Even numbers evenly, are suche numbers as maie be parted continually into even halves, till you come to an unitie. As for example, 32. first is divided into 16, as his even halfe: and again, 16 into 8, as his halfe: and 8 againe by 4, is parted into 2, even partes: Then 4, also by 2, and that 2, is divided into 2, unities, as his same halves.

Even numbers unevenly, are suche numbers as maie be divided into 2, equal partes: which are odd numbers. As, 18 is divided into 9, and 9, as his halves, and the are odd. So, 10 is divided by 5. And 30 by 15, with a great number more of suche sorte.

Numbers even evenly and oddly, bee commonly called suche numbers, as maie be divided into 2, equal partes and even halves; but before you come to an unitie, the halves will be odd numbers. As, 60, maye be first parted into 30, and 30, which are even: And the againe divided by 15, which is odd.

Likewise, 24 is divided first by 12. And that 12, by 6, and lastly, 6 is divided by 3, which is an odd number.

So, 28, maie be divided into 2, equalle and even halves,
halues, that is into 14, and that 14 into 7, which is the halfe of 14, but is an odd number.

Scholar. This I perceive well. And, as I judge, the distinction into those 3 kinds, is not only reasonable, but also needful. And yet you seem to speake doubtfully, of this last member. Because I remember not that you use this word commonly, but where you give place rather to custom, then to reason.

Haller. Diets to custome of the common sorte of writers, rather then to the judgemeute of the most anciente writers.

And so in this case Euclide doeth not seeme to admitte this thirde member. But he compteth it under the second kind. As male well appere in his 9 boke, and 34 proposition, where he calleth suche a number, evenly even, and unevenly odd also, which place cōserved with the definitions in the same booke, doeth approue in many wise mennes opinions, that Euclide minded but 2 only kindes of those numbers. And yet in this thing (I think) he did rather approue 3. varietie by his propositions, then establishe onely 2. sortes by his first definitions.

But herein I will spende no more tyme. But sake brely that the distinctiō of 3. kindes, serveth to good use, and ease in teacheing.

And now for further knowledge of numbers, some are called numbers perfecte, some are numbers imperfect.

Perfect numbers are suche ones, whose partes togued together, will make exactly the whole number. And therefore are 6, and 28, accounted perfecte numbers: because the partes of eche of them added together, doe make the sul and intere number, whose partes thei bee. As of 6, the halfe is 3, the thirde parte is 2, the sixte parte is 1. As for a quarter, and fife parte it hath not in whole number. Now put together, 1, 2, and 3, and thei make fife 6, whose partes thei bee.

And
The seconde parte

And therefore is 6 a perfecte number.

Likewaies, 28. hath for his halfe. 14. for his quarter. 7. for his seventh parte. 4. and for his souwesth parte. 2. and for his 28. parte. 1. all which put together, that is. 1.2.4.7. and 14. do make 28. of this sort there are very fewe more in comparis, And soz an example, I sett here, as many as are under. 6000000000. and the are these 6. 28. 496. 8128. 130816. 2096128.

But now of the contrary kind, imperfectenombres be suche, whose partes added together, do make either more or lesse, then the whole number it selfe: whose partes thei bee.

And if the partes make more then the whole number, then is that number called superficuose, or abundaunt. As 12. whose partes are. 1.2.3.4.6. which make 16.

So. 20. hath for his partes. 1. 2. 4. 5. 10. which make 22. Likewaies. 120. hath these partes.

1. 2. 3. 4. 5. 6. 8. 10. 15. which make 240.

And if the partes make lesse then the whole number, whose partes thei be, then is that number called Diminute, or defectuose. As 8. hath these partes. 1. 2. 4. which make but. 7.

So. 16. hath these partes. 8. 4. 2. 1. and thei make only. 15.

Likewaies. 32. whose partes are. 1. 2. 4. 8. 16. and make but. 31.

Scholar. In all these numbers I note that you reckon one, soz a parte of eche one of theim: which befoze I thought you had denied.

Master. I. cannot neither multiplye noz divide, and therforee compoundeth no number. But one make increase addition, and therefore where partes be added together, there, 1. make well be called a parte.

And this shall suffice foz the division of curnumbers.
of Arithmetike.

bers Abstracte.

Now to speake of odde numbers, some of the arc cum. Odden bers pounde, some uncompoide. They are compounde, which compound. mate bee divided into any other partes then unities. As. 9, which is compound of 3. And. 15, that is made of 5 and 3. Also. 21, is compounde of 7 and 3. And so furthe. But. 3, 5, 7, 11, 13, 17, 19, 23, 29, and suche like, bee odde numbers uncompoide. For they are not uncompoide, made of any other then of unities.

Here must you understande by composition, the multiplication of the partes of numbers together, as you remembred, before was declared.

Scholar. I consider it so. And I remembred all that you have taught me, for the division of numbers abstracte and absolute. What sate you now of numbers relative? Numbers

Master. Some times their relation hath regard unto their partes, namely, whether these 3, that bee so compared, have any common parte, that will divide theim bothe. For if thei have so, then are thei called numbers commensurable. As. 12, and. 21, bee numbers commensurable; fo2. 3, will divide ebe of theim.

Likewates. 20, and. 36, bee commensurable, sypng 4, is a commo division fo2 theim bothe. But if thei have no suche common division, then are thei called incommensurable. As 18 and 25. For 25 can bee divided by no nom: Incommensurable more then by. 5. And. 18, can not be divided by it, surable.

In like maner, 36, and. 49, are incommensurable: For 49, hath no division, but. 7, And 7, can not divide. 36.

Scholar. Doe you meaneth then, that incommensurable numbers, have no coparison nor proportion together?

Master. No, nothing lesse. 3 or any 2 numbers mate have coparison 1 proportion together, although they be incommensurable. As. 3, and. 4, are incommensurable, and yet are they in a proportion together: as shall appeare anon.

But first I will declare unto you, the varieties of
The seconde parte

Proportion. proportion, where in there maie be double conference: I
meane of the letter to the greater, or of the greater to
the letter.

Whi the greater is compared to the letter, it is called
a proportion of the greater inequality. As 6 to 2, 2 2 to 3.
And when the letter is conferred to the greater, it
is called a proportion of the lesser inequality. As, 3, to 5,
2, 2 to 6.

Scholar. And what if I would compare two equalle
numbers together?

Master. That is accounted also a proportion of
many men: and is called the proportion of equalitie. And
then ought the first division of proportion to be, thus

Equalitie.

Proportion of

Inequalitie. The greater.

The letter.

So proportion of the greater inequality, is divided
into 5 severall kindes: whereof, 3, be simple, and 2 o-
ther compound. The firste kind is, when a greater
number containeth the lesser diverse times: as twise,
or thrice, or of tene once. So, 6, containeth, 3, twise; and
it containeth, 2, thrice. This proportion is called
genearly, multiplex, that is to saie, many fold: but
specially it is named, according to the tymes that it
conteineth the letter. So that if it contain hym twise,
then is it named dupla, or double. As 2 to 1 and 4 to 2.

And if it containe it thrise, As 3 to 1 and 6 to 2, it
is called tripla, or triple.

If it containe it, 4, tymes, then is it quadrupla, or
quadruple.

Of these and of diverse other sortes in this kind al-
so, here are the names briefly set downe with eraples.
of Arithmetike.

| Quadrupla. | 4:to:1:8:to:2:12:to:3:16:to:4 | Fourthfold. |
| Quintupla. | 5:to:1:10:to:2:15:to:3:20:to:4 | Fivefold. |
| Duodecupla. | 12:to:1:24:to:2:36:to:3 | Twelfold. |

And so infinitely.

Beside this there is an other kinde of proportion, when the greater containeth the lesser, more then ones, and not twice: and that make him 2 sortes. For if the greater containeth the lesser, and any one parte of him, that proportion is called Superparticular.

For example, take 5:to:4. Sith, 5:doth containe 4, and his quarter. Likewise, 6:to:5, is in the same kinde of proportion: although, not of the same speciall sorte. For 6:comprehendeth 5, and his five parte.

So that so a more speciall distinction, eche of these and many other, have their several names, according to that parte, whereby they do containe. As if it containe the halfe more, it is named Sesquialtera. In which the proportion are these numbers following.


But if the greater comprehendeth the lesser, and his thirde parte, then is that named Sesquitercia proportion. As in these.


And when the fift, firt, sixt, seuenth, or eight part dooth make the proportion, or any other part els, the name is taken of that same parte. As soz brieſznelce I will here sette examples.

|||\[\text{Sesquiquarta.}\]
The seconde parte

Sesquiquarta. 5. to. 4: 10. to. 8: 15. to. 12. 1: A quarter more.
Sesquiquinta. 6. to. 5: 12. to. 10: 18. to. 15. 1: A siste more.
Sesquisexta. 7. to. 6: 14. to. 12: 21. to. 18. 1: A siste more.
Sesquiocta. 9. to. 8: 18. to. 16: 27. to. 24. 1: An eight more.
Sesquonona. 10. to. 9: 20. to. 18: 30. to. 27. 1: A ninth more.
Sesquidecima. 11. to. 10: 22. to. 20: 33. to. 30. 1: A tenth more.
Sesquidecima. 12. to. 11: 24. to. 22: 36. to. 33. 1: A leuenth more.

And so as farre as you lists to procede in suche proportion: where one parte of the lesser, is the tylfe difference and exceede, betwene it and the greater.

But if the difference be 2, partes. 3, partes, o2 more

Superparties partes: the proportion is named superpartienae. As. 5. to 3. And. 10. to. 6. For as. 5. containeth. 3. and. 1/3. of it: so 10. holdeth. 6. and. 1/3. of it.

Scholar. Now I perceive some bye also, of the distinction betwene a parte and partes in number: Of whiche at the beginnyng you did speake. But how many kindes are there of this sorte?

Master. There are infinite kindes in this sorte of proportion, as well as in the other. But for examples sake, I will set furthe some of the mosste common nombers: that therby you maye gather the forms of the reste. And these be the, with their names.

<table>
<thead>
<tr>
<th>Tertia</th>
<th>5. to. 3: 10. to. 6: 15. to. 9: 20. to. 12: 21.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonae</td>
<td>11. to. 9: 22. to. 18: 33. to. 27: 44. to. 36: 39.</td>
</tr>
</tbody>
</table>
of Arithmetike.

| Superintepartis: | Quartas. 7 to. 4. 14 to. 8:21 to. 12:28 to. 16. |
|                | Quintas. 8 to. 5:16 to. 12:24 to. 15:22 to. 20. |
|                | Septimas. 10 to. 6:20 to. 14:36 to. 21:42 to. 28. |
|                | Octauas. 11 to. 7:22 to. 15:38 to. 22:44 to. 28. |
|                | Decimas. 13 to. 8:26 to. 16:42 to. 26:52 to. 36. |
|                | Vndecimas. 14 to. 9:28 to. 18:44 to. 28:52 to. 40. |
|                | Decimastertias. 16 to. 10:32 to. 20:48 to. 30:52 to. 40. |
|                | Decimasquartas. 17 to. 11:34 to. 22:50 to. 42:60 to. 56. |
| Superquadrapartiens: | Quintes. 9 to. 5:18 to. 11:27 to. 15:36 to. 20. |
|                  | Septimas. 11 to. 6:22 to. 14:36 to. 21:44 to. 28. |
|                  | Nonas. 13 to. 7:26 to. 16:42 to. 26:52 to. 36. |
|                  | Vndecimas. 15 to. 8:30 to. 20:48 to. 30:52 to. 40. |
|                  | Decimastertias. 17 to. 9:34 to. 22:52 to. 36:60 to. 56. |
|                  | Decimasquartas. 19 to. 10:38 to. 30:56 to. 42:60 to. 56. |
| Superquintupartiens: | Sextas. 11 to. 6:22 to. 12:33 to. 18:44 to. 24. |
|                 | Septimas. 12 to. 7:24 to. 14:36 to. 21:44 to. 28. |
|                 | Octauas. 13 to. 8:26 to. 16:39 to. 24:42 to. 27. |
|                 | Nonas. 14 to. 9:28 to. 18:42 to. 27:51 to. 39. |
|                 | Vndecimas. 16 to. 11:32 to. 22:43 to. 33:51 to. 45. |
|                 | Duodecimas. 17 to. 12:34 to. 24:51 to. 36:60 to. 56. |
|                 | Decimas tertias. 18 to. 13:36 to. 26:54 to. 39:60 to. 56. |
|                 | Decimas quartas. 19 to. 14:38 to. 28:57 to. 42:60 to. 56. |
|                 | Decimas sextas. 21 to. 16:42 to. 32:63 to. 48:60 to. 56. |
| Supersexstudpartiens: | Septimas. 13 to. 7:26 to. 14:39 to. 21:51 to. 33. |
|                  | Vndecimas. 17 to. 11:34 to. 22:51 to. 33:60 to. 56. |
|                  | Decimastertias. 19 to. 13:38 to. 25:57 to. 39:60 to. 56. |
|                  | Decimas septimas. 23 to. 17:46 to. 34:69 to. 51:60 to. 56. |
|                  | Decimas nonas. 25 to. 19:50 to. 38:75 to. 57:60 to. 56. |
|                  | Vicesmas tertias. 29 to. 23:58 to. 46:78 to. 69:60 to. 56. |

Scholar. I understand by these examples, somewhat of their reasons: but I perceive you do not follow their natural order, without interruption, in these.
The seconde parte

these of the lasse kinde.

Master. To thinte ne you maie the better under- 
stande good ground in that omission, I wil set furthe 
here those omitted nombers: That you maie see how 
thei would expresse some other propozition here named 
And thersoe thei doe seime rather to be omitted, then 
in deede so to be. 
Marke theim well.

\[
\begin{align*}
\text{Secundas.} & : 4. \text{to.} 2 : 8. \text{to.} 4. & 2 \frac{1}{3} \\
\text{Quartas.} & : 6. \text{to.} 4 : 12. \text{to.} 8. & 1 \frac{1}{2} \\
\text{Superbipartiens.} & : 8. \text{to.} 6 : 16. \text{to.} 12. & 1 \frac{1}{2} \\
\text{Sextas.} & : 10. \text{to.} 8 : 20. \text{to.} 16. & 1 \frac{1}{4} \\
\text{Ottanas.} & : 12. \text{to.} 10 : 24. \text{to.} 20. & 1 \frac{1}{4}
\end{align*}
\]

Scholar. In deede here I see, the firste is double 
propozition. The seconde sesquisexta, the thirde sesqui- 
sexta, the fowrthe sesquiquarta, & the fiftte sesquiquinta. 
Mater. So marke these other.

\[
\begin{align*}
\text{Secundas.} & : 5. \text{to.} 2 : 10. \text{to.} 4. & 2 \frac{2}{3} \\
\text{Tertias.} & : 6. \text{to.} 3 : 12. \text{to.} 6. & 1 \frac{1}{2} \\
\text{Supertripartiens.} & : 9. \text{to.} 6 : 18. \text{to.} 12. & 1 \frac{1}{2} \\
\text{Sextas.} & : 12. \text{to.} 9 : 24. \text{to.} 18. & 1 \frac{1}{2} \\
\text{Nonas.} & : 15. \text{to.} 12 : 30. \text{to.} 24. & 1 \frac{1}{2}
\end{align*}
\]

Scholar. The firste of these I knewe not, but all 
the other are named before. 
Mater. The firste is a compounde propozition(as 
anon I will declare) and is named: duplica sesquisexta. 
But now wil I sitte furthe al the other omitted 
names.

Secundas.
of Arithmetike.

<table>
<thead>
<tr>
<th>Secundas</th>
<th>6: to 2:12: to 4</th>
<th>Tripla.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tertias</td>
<td>7: to 3:14: to 6</td>
<td>Dupla sesquiteria.</td>
</tr>
<tr>
<td>Quartas</td>
<td>8: to 4:16: to 8</td>
<td>Dupla.</td>
</tr>
<tr>
<td>Sextas</td>
<td>10: to 6:20: to 12</td>
<td>Superbipartien tertias</td>
</tr>
<tr>
<td>Octauas</td>
<td>12: to 8:24: to 16</td>
<td>Sesquialtera.</td>
</tr>
<tr>
<td>Decimas</td>
<td>14: to 10:28: to 20</td>
<td>Superbipartien quintas</td>
</tr>
<tr>
<td>Duodecimas</td>
<td>16: to 12:32: to 24</td>
<td>Sesquiteria.</td>
</tr>
<tr>
<td>Decimas quartas</td>
<td>18: to 14:36: to 28</td>
<td>Superbipartien septimas</td>
</tr>
</tbody>
</table>

Superquadripartiens.

<table>
<thead>
<tr>
<th>Secundas</th>
<th>7: to 2:14: to 4</th>
<th>3\frac{1}{2} Tripla sesquialtera.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tertias</td>
<td>8: to 3:16: to 6</td>
<td>2\frac{3}{4} Dupla superbipartien tertias.</td>
</tr>
<tr>
<td>Quartas</td>
<td>9: to 4:18: to 8</td>
<td>2\frac{3}{4} Dupla sesquiquarta.</td>
</tr>
<tr>
<td>Quintas</td>
<td>10: to 5:20: to 10</td>
<td>Dupla.</td>
</tr>
<tr>
<td>Decimas</td>
<td>15: to 10:30: to 20</td>
<td>Sesquialtera.</td>
</tr>
<tr>
<td>Decimas quintas</td>
<td>20: to 15:40: to 30</td>
<td>Sesquiteria.</td>
</tr>
</tbody>
</table>

Superquinqupartiens.

<table>
<thead>
<tr>
<th>Secundas</th>
<th>8: to 2:16: to 4</th>
<th>Quadrupla.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tertias</td>
<td>9: to 3:18: to 6</td>
<td>Tripla.</td>
</tr>
<tr>
<td>Quartas</td>
<td>10: to 4:20: to 8</td>
<td>Dupla sesquialtera</td>
</tr>
<tr>
<td>Quintas</td>
<td>11: to 5:22: to 10</td>
<td>Dupla sesquiquinta.</td>
</tr>
<tr>
<td>Sextas</td>
<td>12: to 6:24: to 12</td>
<td>Dupla.</td>
</tr>
<tr>
<td>Octauas</td>
<td>14: to 8:28: to 16</td>
<td>Superbipartien quartas.</td>
</tr>
<tr>
<td>Nonas</td>
<td>15: to 9:30: to 18</td>
<td>Superbipartien tertias.</td>
</tr>
<tr>
<td>Decimas</td>
<td>16: to 10:32: to 20</td>
<td>Superbipartien quintas.</td>
</tr>
<tr>
<td>Duodecimas</td>
<td>18: to 12:36: to 24</td>
<td>Sesquialtera.</td>
</tr>
<tr>
<td>Decimas quartas</td>
<td>20: to 14:40: to 28</td>
<td>Superbipartien septimas.</td>
</tr>
<tr>
<td>Decimas quintas</td>
<td>21: to 15:42: to 30</td>
<td>Superbipartien quintas.</td>
</tr>
<tr>
<td>Decimas sextas</td>
<td>22: to 16:44: to 32</td>
<td>Superbipartien octauas.</td>
</tr>
<tr>
<td>Vicecycles</td>
<td>26: to 20:52: to 40</td>
<td>Superbipartien decimas.</td>
</tr>
</tbody>
</table>

Scholar. I see well that these proportions, bee agreeable with some other name: and therefore might some superfluous in this place.

Walter.
The seconde parte

Master. Not onely superfluously, but also falsely should the be placed here: seynge thet doe belong to other places of right.

Scholar. Why doe you not name theim all by Englishe names?

Master. Because there are no suche names in the Englishe tongue. And if I should gie theim newe names, many would make a quarrelle against me, so obscuring the olde Arte with newe names: as some in other cases all redy have done.

Scholar. Yet I pracie you declare those doubtfull names of compundee proportions.

Master. As there is one kinde of proportion, that is named multiplex, or manysolde, whiche dooth containe the lesser diverse times exactly. And two other whiche do containe the lesser ones, and some parte of partes of thesame: So those kindes maie be compounded together. As when the greater number containeth the lesser, twise, or thwise, or oftener: and yet more over some parte of partes of thesame. So. 8. containeth 3 twise, and his \( \frac{1}{3} \). And 10 comprehendeth 3, thwise and his \( \frac{1}{3} \).

The first example is generally called multiplex superpartiens: because the greater containeth the lesser certaine tymes, and some partes of it besides. But more spacially it is called dupla superbipartiens tertias, that is, double with \( \frac{1}{3} \) more.

The seconde example is generally referred to multiplex superparticularis: because in it the greater comprehendeth the lesser oftentymes, (as here thwise) and his \( \frac{1}{3} \) more. And therefore spacially it is called tripla sesquequitertia.

But as I doe intende hyesly to ouer runne this parte: so will I by tables set for the the kindes of the with their examples.
of Arithmetike.

The table of proportion of the greater inequality.

<table>
<thead>
<tr>
<th>Manyfoldes</th>
<th>Supparticulare</th>
<th>Supparticulare</th>
</tr>
</thead>
</table>

C. i. Examples
The seconde parte

Examples of eche compounde kinde, mentioned in the former table.

**Manifolde Superparticulare.**

- **Double.**
  - Sesquialter.
  - Sesquiquarte. 9 to 4.

- **Triple.**
  - Sesquitierce.
  - Sesquiquint. 16 to 5.

- **Quadriple.**
  - Sesquialter.
  - Sesquisext. 25 to 6.

**Manifolde Supparticulae.**

- **Double.**
  - Superbipartient tierces.
  - Superbipartient quartas 11 to 4.

- **Triple.**
  - Superbipartient tierces.
  - Supertripartient quartes. 15 to 4.

- **Quadriple.**
  - Superquintupartient quartas 29 to 6
  - Supersexupartient septuasmas 54 to 7

**Scholar.** What more is there to bee learned of these proportions: For by these formes, I maie easilly gather the value or rate of any proportion.

**Master.** This maie stande for their numeration: saue that moste aptly the ought to bee sette as fractions, in their leaste tearme: as you haue here diverse examples.

**Scholar.** You meane that double sesquialter must be written thus $\frac{1}{3}$, and so of the rest.

**Master.** Oyes thus $\frac{2}{3}$, and so triple sesquiquint in this sorte; $\frac{3}{5}$, $\frac{1}{2}$ thus $\frac{5}{8}$ and so of all other.

And so further woeske, you shal understande that proportions maie bee added, subtracted, multiplied and divided; and verie straunge woeske therby achieued:
of Arithmetike.

achieved. For of the Arte of Proportions, dependeth all the subtleties, and fine workes, not onely of Arithmetike, but also of Geometrie; besides farther matter that as now I will not touche. But as for the workes of Proportions, I will omitte them in another time: considering not only the troublesome condition, of my unquiet estate; but also the convenient order of teaching, whereby it is required that the extraction of roots, should go orderly before the arte of Proportions: which without those other, can not be wrought.

Therefore will I now onely declare these kinds of proportion, which yet are not spoken of: to the intent that you may have here, the general division of numbers, somewhat sufficiently touched.

As you see that betwene any two numbers, there must be a conference of proportion: so if any one proportion be continued in more then 2 numbers, there must be then a conference also of these proportions, in their severall terms: and that conference or comparison is named Analogie: which some delight to call proportionalitie: As in this example, where numbers beare like proportion in their progression: 4, 6, 9. You see that 6 to 4, is in proportion sequaler: and so is 9 to 6, and therefore is there a like proportion betwene the 2 laste, as there is betwene the 2 firste.

Scholar. This I consider well by progression in Arithmetike.

Master. Likewise where fouer termes or more be set in order of proportion, as here 2, 6, 18, 54.

Scholar. I perceiue this well: for here the proportion is triple. But what saye you to this foure of comparison in Proportion: As 6, is to 2: as 30 to 10. Is it not all one kind of Analogie?

Master. It is one kind of Analogie generalle, whiche may be called diriecte Analogie: because the first direcste one is compared to him that doth folowe nexte: so eche logie.

C.1. other
The seconde parte

other is still referred to that, that foloweth nexte. But this is the difference: that in the firste, there is a continuance of collation: and one terme is compared with twoo numbers: But in that foume of example, whiche you put, there is no number compared twice: for the first is referred to the seconde, and the second to the thirde. And so have the seueralle names to distince them a sonder.

Wherefoze whe the first number is referred to the seconde, and that seconde to the thirde: the proporzione is called continualle: and it make consiste betweene 3 terme. As 5, 15, 45. doe proceade in a continual triple proportion. For as 5, is to 15:so is 15: to 45.as you doe see. But when I saie thus: as 5, is to 15:so is 6, is to 18. Here is a triple proportion, but not continualle. For the seconde terme beyng 15, is not compared with the thirde terme, that is 6. And therefore is it called a proportion discontinualle.

Scholar. Now I perceive certeinly their distincction: for in twoo points these examples doe agree, and differ in a thirde pointe.

Firste thei agree in that(as you said)that the fozmoiste is referred to the other that foloweth it nexte: And secondly, thei agree in this also, that bothe are compared in a triple proportion. But in this thei differ, that the seconde terme, doeth not bcare like proportion to the thirde, as the thirde doeth to the fourth 02 the firste to the seconde.

Master. Farther moze there is to be noted, that in discontinualle proportion, there can bbe no fewer then sower terme, 02 numbers: and so by even fozmes still, as 6. 02. 8. and so so2. Where as in continual proportion, your terme maie bbe of any number, even 02 odd above.2.

And although I might saie moze of the diversities of proportion:as of Proportion conversed 02 indiretly, Proportion,
of Arithmetike.

portion interchaunged, compounde Proportion, parted Proportion, reversed Proportion, and Proportion by equalitie. Yet I think better to proceed for this time, to the other kindes of number, and to reserve the explication of proportions to their peculiare place.

Scholar. As you knowe the best order, so it shalbe mete that you doe use your owne judgement therein.

Of figuralle numbers.

Master.

The nerte kinde of numbers are called Figuralle numbers: because they doe, or maie represente some figure: And are ever considered in relation to those formes.

Some of them have a comparison and relation to length onely, and therefore are named Linearie numbers: which name, although it maie Nombers li; bee referred moste aptly to suche numbers, as will appear. Make no other forme duely, yet maie it also be applied to any number abstract. With all suche numbers maie be considered as the sides of other figuralle numbers.

Secondly, numbers maie be considered, according to suche formes as their make other in progression, or in multiplication: And those maie well be named Superficiall numbers, or Flattened numbers. Whereof there bee Superficiall as many varietie, as there bee diversities of figures numbers. In Geometrie. As numbers Triangular, Quadrat, Cinke, Flattened, Sifanealed, and so furthe. Also numbers circu: bers, lare, diametralle, & like flattes, all whiche numbers haue bothe lengthe and breading: and thereof bee named Superficiall numbers.

C.lij. Beside
The seconde parte

Beside these there are other numbers, which are made of many multiplications, and they are called founde numbers: because that as by the firste multiplication, they take lengthe and breadth, like flatte numbers, so by the second multiplication, they take depth also: And therefore be they named bodily numbers, or sound numbers.

The leaste of them all is commonly called a Cube, or a Cubike number: And the other in their degrees severally named, as they bee made by severalle multiplications. For according to the number of their multiplications, they take their names. And all that have like number of multiplications, are of one kinde, and bere one name: as well in flatte numbers, as in sounde.

But considerynge the infinite multitude of those figuralle numbers, I thinke beste to speake of them onely in this place, whiche have muche profitable use in this arte. And, of those, among infinite flatte numbers, I will take onely soower: That is to saie, square numbers, longe squares, diametralle numbers, and like flattes.

Square numbers are those, whiche maye be diuided by some one number, and have the same number for the quotiente: that is to saie, that a square number is made by the multiplication of any number into it self, as 10 multiplied by 10, maketh, 100. That, 100, is a square number: whiche, 100, if I doe diuide by 10, the quotiente will be, 10, also.

Scholar. So, 4. multiplied by 4, maketh, 16; and that must be a square number by like reason.

Master. So it is.

Scholar. And if I multiplye, 9, by 4, is not that a square number: Seyng soower semeth to make all numbers square by multiplication.

Master. Consider this well, that a square number doeth make suche a square in number, as a inscribed square doeth make in Geometrie: That is suche a one whole
of Arithmetike.

whose sides are equalle. For and if the one side be longer then the other, that figure in Geometrie is called a long square, and so it is named in number, a long square also.

Now if I sette downe the figure of your number, as you termed it, and sette 4. for the one side, and 9. for the other, this will the figure shew.

Where you see a plain longsquare: yet is the whole number that amounteth of this multiplication: truely named a square number, as here you maie see. But then is the side 9; roote 4, neither. 4. nor. 9. but. 6.

Scholar. How I understand it: and the better by this figuralle example. And here also I have learned what a roote is: for you seeme to expounde it, to bee the roote side of a figuralle number.

Master. Every Slater number, and every sounde number also have their sides: But no Slater number, save onely squares have a roote: because a roote in Slater numbers, is a number multiplied by it self.

And in sounde numbers, the onely have rootes, which be made by many multiplications, of some one number by it self: other by that, whiche riseth of it.

As when I saie, twoo tymes, twoo twise, maketh 8. that number is a sounde number: and is named a Cube. And so, 3. tymes, 3. thrise, dooth make, 27. which is also a Cube.

And generally, any number that is made by suche 2. multiplications, is called a Cube, or a Cubike number. A cube.

And the number of that multiplication, whiche commonly is named the multiplier, is in this pointe calleled the Cubike roote of that number.

Wherefor, thus also maie you define a Cubike no: A cubike root, number.
The seconde parte

ber: to bee suche a number, as beeinge divided by his roote, shall have for the quotiente the square of the same roote.

Scholar. Hereby I perceiue, that one multiplication, of any number by it selfe, doeth make a square number. And twoo multiplications in that sorte, doe make a Cubike number.

What if I doe multiplicke any number thysse, ozfower tymes, ozbffower tymes, ozbffower tymes, are there proper names for suche numbers?

Master. Yes in deede: as I will declare anon.

But firste before we attempte the other sounde nombes, it shal bee mete, that we doe declare those twoo sortes of flatte numbers, whiche I named before: that is diametralle numbers, and like flattes.

Adiametrall A Diametralle number, is suche a number as hath twoo partes of that nature; that if thei bee multiplied together, thei will make the said diametralle number;
And the squares of those twoo partes, beeinge added together, will make a square number also; whose roote is the diameter to that diametralle number.

As 12 is named a diametralle number, for that he hath twoo partes, that is, 3. and 4, whiche beeinge multiplied together, doe make 12. that is the firste number.
And if their squares be added together, thei will make a thirde square: and the roote of that number will bee the diameter to that flatte square of 12. As in this example you see.

The one side is 4.

and the other side is 3, whiche bothe multiplied together, doe make 12. Then take the square of lower whiche is 16 and the square of 3, whiche is 9. and put them together
of Arithmetike.

together, and the will make 25. whole roote, beyng 5. is the diameter of that platte some.

Scholar. That doe I perceive well, because it is confirmed by the 33. theorem of the pathwaye.

Mater. Yet take another example. In this platte some of 60. you see the one side to bee 5. and the other side to bee 12. Now take the square number of 12. which is 144. and the square of 5. which is 25. and put them together; so will it make 169. which is a square number: and hath 13. for his roote.

Likewaies 120. is to be accounted a diametralle number. For so muche as it hath twoo partes: that is 8. and 15. which beyng multiplied together, doe make the firste number. 120. And the square of those twoo partes (that is 64. 102. 8. and 225. 102. 15.) beyng bothe added together, doe make 289. which is a square nober: and hath for his roote 17. And therefor that 17. is the diameter to that diametralle number. 120.

Like examples infinite might I give you. But these for explication of the name, maie suffice.

Scholar. I doe well understande the examples: saue that I knowe not how to finde the roote of the laste square number, which amounteth by the addition of the former twoo squares together.

Mater. That arte will I teache you anon. But we maie not forgette firste to ende all the definitions of soche names, as I minde to write of.

Whereof yet there resteth like flater: which maie bee as well taken for triangular figures, as for quadraté figures.

So that of any of them, when the sides of one plat some, heareth like proportion together, as the sides
of any other flatte forme of the same kinde doeth, then are those formes called like flattes. As in these 2. longe 3. Squares: because the sides of them bothe are in one proportion (for 6. is triple to 2. as well as 9. is triple to 3.) Therefore are 2. the whole figures called like flattes.

And so of due convenientie, their numbers (that expresse their quantities, which here are 27. and 12) be called by the like names, like flattes.

Farther more in triangles (as here you see) if the sides of the one beare like proportion together, as the sides of the other do: then are they called like flattes also. And their numbers, that declare their quantities, in like sorte are named like flattes.

Scholar. I perceive here: As 4 is to 2: so 6. is to 3, bothe being in a double proportion. And therefore 6 and 24. are to be called like flattes.

Master. You understand it well.

And thus haue we briefly ouer runne the division of number, into his principalle kindes: And haue set forthe the definiitios of eche of them, with examples.
of Arithmetike.

The use of them you shall be largely in the practice of this art.

But to the intent you may the better observe and regard these two laste kinds of numbers: which are commonly neglected of artes men, I will shew you some use of them, with their properties.

First, all ditametral numbers doe sette for the a triangle, haung all three sides known: which thing as it doth serve to many and wonderful purpuses: so can it be found in no other numbers, then only in ditametral numbers.

For, although in figures Geometrical, you may either more unsallibly finde one line, that will make a square, equall to the twoo squares of any other twoo lines (as in the pathe waie you doe see it taught) yet the measure certaine of those sides, are not known.

Whersoever in number that is not possible alwaies to be done: neither can it be done with any other no- bers, then only ditametrical numbers. Yet maie other numbers goe very nigh, as namely in these examples of square numbers: whose double, I take for the squares.
The seconde parte

res of the sides, because thei are equall: and thei make. 8, 50, 288, 1682, 9800, 57121, 332928. All whiche differ onely by an unitie, from a square nomber.

For nine is a square nomber and so are these other folowyng.

49, 289, 1681, 9801, 57121, 332929.

Whiche examples if you doe consider well hereafter, they will helpe you to gessse at the nighste rootes of nombers that be not square. And also for doblyng of squares, in a square forme: within an unspeakeable nere Ness.

For as in doblyng of this greater square, 166464, there riseth, 332928, whiche wanteth one of a inste square. You se easely, that as that one is but a smalle portion to the whole square: So yet, that one wanteth not in the roote, but in the whole square: where by you maie perceive, that it is a very smalle and un- sensible parte of one, that wanteth in the roote.

Scholar. It must seene by reason of multiplicati-
on; that it is scarce the. 10000, parte of one.

M aster. You saye truthe.

Scholar. But how shall I finde the diameter of suche nombers?

M aster. That is easely done, if you knowe firste certainely that your nomber is a diametrall nomber.

And seconderly, if you knowe the true partes of it:
of Arithmetike.

it: whiche you should vsue in this case.

Scholar. Will not any twoo suche partes serve,
whiche by multiplication will make the whole nom-
ber?

Master. You maye by the foreuer examples, easily
se the contrary. For 12 is a diametrall number: and hath
these partes (as it is sone perceiued). 2, 3, 4, 6. Yet if
you take 2, and 3, 6, for the sides of it, they will not
make a diameter in knowne number.

Scholar. That I understand: for the square of 2.
being, 4, added to, 36, whiche is the square of 6, doeth
make, 40. Whose roote must be greater then 6, and
lesse then 7. And therefore, 40, can have no roote in
whole number.

Master. Neither yet in brooken numbers: for that
is a generall rule: that if any whole number have a
roote, that roote shall be a whole number. So that if
the roote can not bee founde in whole number: you
shalt never finde it in brooken numbers.

And for more certaintie of that I saied before, that
all partes be not apte for the sides of a diametrall nom:
ber, to finde out the diameter: marke well the seconde
example, whiche is 60, and hath these partes.

2, 3, 4, 5, 6, 10, 12, 15, 20, 30.

So that beginnyng with the two extreme, that
is, 2, and, 30, thei will by multiplication make, 60.

And likewaies any two nombers, equally distant
from those extremes: As, 3, and 20, likewaies, 4, and
15: other, 5, and, 12. And in like maner, 6, and, 10. All
those couples by multiplication doe make, 60. Yet
none of them are apte sides to finde the diameter by,
but only 5 and, 12. For of the other sides being multi-
plied squarly (that is by the selfes) and those squa-
res being added together, there will not rise a square
number. As you shall better understand, when you

D.i., have
The seconde parte

hane learned to knowe square nombers, by extractio
of their rootes.
Yet in the meane reason I will set for the certaine
notes, to knowe the diameter, and the apte sides, in all
diametralle nombers.

1. And firste I saie, that as thei are three nombers in
all (I meane the twoo sides, and the diameter) so al
waies if the firste o2 lease side bee odde, then shall
there be twoo of them odde nombers. And the diame-
ter shall ever bee the other of the odde nombers: that
is to saie, the greateste of them.

2. Secondarily. It is true that all diametrall nombers
are euen nombers. And no odde nomber can bee a di-
ametralle nomber.

3. Thirdly, I saie, that all odde nombers above one,
maie be the lesser side in doche diametralle nombers.
But euen nombers doe not serue so generally: for
thei onely maie stand in doche place, whiche be grea-
ter then. 4: As. 6, 8, 10, 12, 14, 16, 18, 20, &c. And none
other euen nombers then doche as maie be dividied by
4. maie be the greater side in any diametralle nomber.

4. Fourthly. If the lesser side bee an odde nomber,
them ordinarily, the square of it is suche equall with
that that amounteth by the addition of the diameter,
to the greater nomber. As in the firste example, 3. is
the lesser nomber, and 4. is the greater: unto them
bothe the diameter is 5. Now 3. hath for his square
9, and so moche is made by the addition of 4. and 5.

Again in the seconde example, the lesser nomber is
5, and his square is 25. The greater nomber is 12, and
the diameter. 13. Put 12. and 13, together, and thei
make 25, whiche is equall with the square of the
lesser.

Like waies, 7. and 24, multiplie together maketh
168. whiche is a diametralle nomber. And because the
square of the lesser side (whiche here is 49.) must bee
equalle
of Arithmetike.

equalle to the greater side, and the diameter added together: therefore seyng 25, added to 24, makest 49. that 25. must nede bee the diameter to the forefaile number.

By these rules (if you doe marke them well) you maye sone perceiue, how to make any diametralle number: if the lesser side be given unto you, and be an odd number. Yet for your case, I will giue you this plaine rule.

When any odd number is propounded: as the lesser side of a diametralle number, and you would finde the other side, and the diameter also: o2 els the diametralle number, that may haue the other side: multiply that propounded number by it selfe, and it will make a square number, and will be an odd number: so that of it you shall finde no suche halfe. Therfore take you those twoo numbers, that are nerte unto the halfe of it: The lesser shall alwayes bee an even number, and shall be the seconde side of the diametralle number: The other number whiche is the greater, shall alwayes be an odd number: and shall bee the diameter of that number whiche you desire. For example marke wel these formes that doe folowe.

If there bee propounded as the one side of a diametralle number: And you would knowe, what maye bee the other side: and what is the diametralle number: And thirdly, what is the diameter to that number: Doe, as I saide before: multiply 3. by it self, and it will make 9. which is a square number, and an odd number: and therefore hath no suche halfe. But the highest numbers to the halfe, are 4. and 5.

Therfore I saie, that 4. which is the lesser of the twoo, is the second side of the diametralle number: and 5. beynge the greater of them, is the diameter it self.

Scholar. Now is it light enoue to perceiue that the diametralle number is. 12: seyng 3. multiplied be lower
The seconde parte

4. maketh. 12.

Master. So is it.

Again, if 5. be assigned for one side of a diametralle
number, and you obturc the former worke you maie
easily finde the other side, and the diameter.

First you see, that the square of 5. is 25. and it hath
no halfe. But 12. and 13. are the 2. numberes nightest
his halfe: wherfoze 12. shall be the seconde side; and
13. must be the diameter. And the diametralle nöber is 60.

Likewaies 7. be set for the lesser side, the grea-
ter side shall be 24. and the diameter 25.

Scholar. Touching this I neede no more instruction: the thyng is so manifest.

Master. Then thewe your knowlege by an exam-
ple, 02 twoo.

And first I appoynte 9 for the lesser side of a diame-
trall number, whereunto I would have you to assigne
the other side, and the diameter.

Scholar. I followe your precepte, and multyli-
9, by it selfe, whereof commeth. 81. whose halfe is be-
twene. 40. and 41. Wherfoze must. 40. be the other
side; and 41. the diameter. And here the diametralle nöber
is. 360.

Master. Prove the like: where. 15. is the lesser
number.

Scholar. 15. multylied square maketh. 225: whose
nightest halfe are. 112. and. 113. of whiche the first is
the seconde side, and the later is the diameter; and the
diametralle number is. 1680.

Master. What shall be the othe numberes: where
21. is the lesser side.

Scholar. 21. peldeth in square. 441. whose pos-
tions nightest his halfe, are. 220. and. 221. And so ap-
nereth their offices, and the diametralle number is 4620

Master. So maie you sale that unto. 27. being the
lesser side; the greater side shall be. 364. and the dia-
eter
of Arithmetike.

Ser. 365. because the square of 27. is. 729. And the diametral number is. 9828.

Scholar. So must it be, by your rule.

Master. Not only the rule doth teach you that it is so, but also the nature and figure of soche flatte numbers. As here you see.

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But to the intente you maie the better understand the nature of these numbers: I wil set for the heere the like sides with other numbers: whereby you maie knowe, that one side maie serue to diverse diametralle numbers.
The seconde parte

numbers. Therefore mark these forms well.

12.

9.

108.

20.

15.

25.

300

28.

36.

15.

39

540.

28.

39.

588.
of Arithmetike.

Scholar. Here I see the same four numbers, 9, 15, 21, and 27, set as the lesser sides: And their greater sides are so that as disagree from the preceding rule. And in 15, 21, and 27, I see two varieties, unlike to the former example. But seeing the sides do disagree, I do not marvel that the diametralle numbers are diverse from the former.

Master. Examine these numbers, whether they be true.

Scholar. I must multiply each side by itself, and then add the together; and if they make as moche ili:ly, as the diameter being multiplied square, then are they true numbers. So I see, that 9, make 81, and 12 doeth yeale 144 whiche bothe added doe make 225. And so moche doth 15 make, being multiplied square.

Likewailles, for the second figure 15, bynge forth E. i. 225.
The seconde parte

225. and 20. giueth 400. that is by addition 625. which somme doeth amounte also when. 25. is multiplied square.

The thirde figure hath 15. also for the one side, whose square is 225. and for the other side 36. which maketh in square 1296. And the bothe together giue 1521. And so many commeth of 39 multiplied by itself in square.

Again for the fourthe figure 21. maketh 441. and 28. doeth yeilde 784. which bothe beyng added doe amounte unto 1225. And so moche doeth there arise by 35. multiplied into it self.

The fift figure hath 21. also and his square is 441. and the seconde side beyng 72. maketh in square 5184. So that bothe those squares doe make 5625. And the like number is made by 75. multiplied in square forme.

Now in the sirt figure 27 beyng multiplied square beyngeth for the 729. And 36. likewise multiplied doeth make 1296. and that with the other will make by addition 2025. which somne (as is well seen) doeth come of the multiplication of 45. by it self.

In the seventh figure 27. multiplied square doeth giue 729 and the other side (which is 120.) doeth beyng for the 14400. These bothe together doe make 15129. And the like somme is gathered by the multiplication of 123. squarely.

So that all those figures doe appere true.

But how thei male agree with your foriner rule, I can not see.

Saster. That rule did I make for nöbers uncomponde. For nöbers compunde have not onely in their owne name the bse of that rule, but also thei folowe the somne of those nöbers, of which thei bee compunde.

So 9. beyng compunde of 3. foloweth the somne of
of Arithmeticke.

of. 3. And therefore as 3. hath 4. so 2 to make the second side with hym, so 9. (being the se. 3.) shall have 12. (which is three 4.) for a matche side with hym.

Likewaies. 15. being compounde of 5. and 3. shall haue their somes in the makynge of the diametral numbers. For as 3. hath 4. so 15. (being five tyms 3.) shall have 25. (which is five tyrme 4.) for the second side.

Again, as 5. hath 12. so shall 15. (being three tymes 5.) have 36. that is three tymes 12.) for his second side.

Likewaies. 21. being compounde of 3. and 7. shall have bothe their somes.

And 27. whiche is compounde of 3. and 9. shall have all the varietis of their somes.

Scholar. I see it is esu. so, and that in the diamet. as well as in the second side. But the diametral number dooth varie moche in them.

Master. Ver doe those numbers agree in a mar. yeilous good proportion. For if you doe consider the proportion of bothe the sides in one figure, to bothe the sides in another figure; and add bothe two proportions together, the addition of theim doth make the number that representeth the proportion betwene their two diametral numbers. Whiche thinge I will now onely touche, as brely as may be, to gwe you occasion to make it better hereafter: Sith this place dooth not fully serve for it. As 3. and 4. being the two sides of a diametral number, doe make 12. So if 9. and 12 be the sides of a diametral number, that number must be 9. tyms 12. that is 108. For 9. is triple to 3.; and 12. is triple to 4. And because the addition of proportions, is like the multiplication of fractions, I must multiply, 3. by 3. or: is 1 by 3, whiche is all one, and that will make 9.

Likewaies, if 3. and 4. be taken for the sides of the
The seconde parte

letter number diametralle, and 15. and 36. foe the sides of the greater number: As the letter number shall bee 12. so the greater must be 540. that is 45. tymes 12.

For 15. unto 3. is in a quintuple proportion, and is written thus. and 36. unto 4. is a noncaple proportion, and is written thus. Now if you multiply these numbers together, they will make 45. which declares the proportions of the twoo diametralle numbers.

And so of all the reste, as you make easily consider.

Scholar. I praise you, let me examine one of the in comparison to that firste diametral number 12.

I see that 15 being the lesser side, and 20. the greater side, do make 300. as their diametralle number: and that 300. is 25. tymes so moche as 12. is. Therefore by your saying the proportion of 15. to 3. and of 20. to 4. must make 25. And so it doeth. For each of them is a quintuple proportion. And it is quickly gessed, that 5. multiplied by 5. doeth make 25.

For farther prooffe, I take the diametralle number 1680. whose sides are 15. and 112. First I see, that 15. to 3. beareth a quintuple proportion: and 112. to 4. is as 28. to 1. Therefore I multiply 28. by 5. and it maketh 140. Then if I multiply that number by 12. it will make 1680.

This is a sufficiente trialle for these numbers.

Of even sides.

But of soche diametralle numbers, as have even numbers for their letter side, you have given no rule, neither example, save only of 8. Therefore I praze you tell me, how shal I finde out the diametralle number, with his other side, and the diameter in soche even numbers.

Master. You shall make it square, as you did in the other numbers, that were odd: And of that square you shall take twoo quarters, which you shall alter in soche sorte, that you shall abate 1. fro the one quarter, and put it to the other quarter. And so have you twoo
of Arithmetike.

two numbers, differing only by .2., and bothe beryng odd. The letter of them two, is the greater side of the diametralle number: and the other is the diameter to it. As .8. beryng your letter side, the square of it is 6.4. whole quarter is .1.6. from which I abate. 1., and there reseth. 1.5. and that is the second side. Also I addc. 1. to 16. and it maketh. 17. which is the diameter.

Scholar. This is no thyng harde. As by example I will prove. If .12. beryng the letter side: his square is 1.44. and the quarter of it is.3.6. Then abatynge. 1. I see there will beryc. 3.5. for the other side of the diametralle number. And addyng. 1.1. to.36. it maketh. 37. to be the diame
ter. And if I multiply. 35. by. 1.2. it beryngeth forthe. 420. which is the diametralle number.

Now for proofe of these numbers, I multiply. 1.2. by it selfe, and it maketh. 1.44. Then I multiply the other side, that is. 3.5. by it selfe, and it yeldeth. 1.2.25. Those bothe together doe make. 1.36.9. And beryng .37. multiplied by it selfe, doeth make the same number. Therfor are the all true numbers.

An other example. 10. beryng set for the letter side, I doe multiply it squarely: and there reseth. 1.0. whose quarter is. 2.5. For which I take ( as you taught me). 2.4. and. 2.6. And so the whole diametralle number is. 2.42. For proofe of the other numbers, I take. 1.0. which commeth of. 1. multiplied square, and to it I adde. 576. which is the square to. 2.4. and the bothe doe make. 676. And so muche amounteth by the multiplication of. 26. squarely.

Mister. This maye suffice for this presente: if you marke that the eue numbers have not only one generallie forme, which I did express in the former rule, but also suche as be compoud of any other nob- hers, even or odd: I have the like numbers in propozi
tion, for the greater side, and for their diameter as the numbers haue, of which thei bee compouded. And because
The seconde parte

because I will not staie to long on this matter, I will here set for the diverse varieties of diamentall nombers, whereby you maie gather not onely the true understanding of the soamer rules: But also in theim you maie see other notable conclusions: and strange wozkes of the natures of nombers.

Marke well this table sozme, with the titles oever it; whiche declare the true meanyng of it.

And where you see one nomber in the firste col-
lumpne against two, three, or force in the other co-
lumpnes, you shall understaande that that nomber is the side to so many seuerall nombers diamentalle.

The table of diame-
tralle nombers.
<table>
<thead>
<tr>
<th>The lesser side</th>
<th>The greater side</th>
<th>The diameter</th>
<th>The number of diametralall</th>
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</thead>
<tbody>
<tr>
<td>3</td>
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<td>65</td>
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<td>7800</td>
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<td>255</td>
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<td>76</td>
<td>761</td>
<td>29640</td>
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<td>2000</td>
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<td>3840</td>
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<td>108</td>
<td>202</td>
<td>7920</td>
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<td></td>
<td>359</td>
<td>401</td>
<td>18960</td>
</tr>
</tbody>
</table>
The seconde parte

This table may ye extente infinvely. And these thinges may ye se, as thinges of greate admiratiō.

1. There is no diametrale nomber, but it may be dividēd by 12. Whersōe thet be all euene nombers euene ly and oddel.

2. AGAIN, there is no diametrale nomber, but it endeth in o.m.i.2.0; in.8.

3. Thirdel, there is no diametrale nomber, that can haue any more diameters then one.

4. Yet maie one nomber bee the diameter to diverse other.
   As you se 25, is the diameter to 168, and also to 300.
   So. 65, is the diameter to 1008, and also to 1500.
   Likewaies, 145, is the diameter to 2448, and to 3432.

5. Fiftely: No square nomber can bee a diametrale nomber.

To knowe a diametrale nomber.

Scholar. These propertyes be notable.
But how shall I knowe, when a nomber is pposed, whether it be a diametrale nomber, or not?
Mastēr. In that thyng I finde a tedious travell, by any rules, in those that write of it. But I wil easte you of noche paine theren.

Firste remember the propertyes of those nombers.
And if you have any other figure in the firste place, then o.2.02.8, it is no diametrale nomber.
Secondarily, if it maie not bee dividēd by 12, althouge it ende in one of those 3. figures, it is no diametrale nomber.

Whersōe, if it haue bothe those twoe propertyes (which an infinite multitude of nombers doe want) and be no square nomber (as none be that ende in 2. 08. 08 with oode cyphers) then lette out all the partes of it, in loche forme, that the letter parte doe stande directly ouer those greater partes, which beyng multiplied together, will make the whole nomber.

And
of Arithmetike.

And then examine those partes, whiche seeme to have any likelihod: according to the former doctrine.
As for example: if 72. be proposed to be examin'd in that sorte, I sette his partes in order thus.

2.  3.  4.  6.  8.
36. 24. 18. 12.  9.

Howbeit I need not to set downe 2. neither 4. soz lesser partes, neither those other greater partes, that answer to them: For, as I said before, they can not be the lesser side in any diametralle nomber. Wherefore they need no examination.

Farthermore, for them that you shall need to examine, if the lesser nomber bee an odde nomber, the square of it must contain double to that greater nomber (that is coupled with it) and one more.

And if the lesser be an even nomber (of them two) that you would examine, then must the square of it containe the greater nomber (that standeth by it), 4. tymes, and 4. more. And this is not only a shorter waie, then I see to be taughte by other artes menne: but it is also moare certaine, for all nombers not compound of other diametralle nombers.

Scholar. By this doctrine it appeareth quickly, that 72. is no diametral nomber.

For although it doth ende in 2. and maie be divided by . 12. yet no couple of nombers here have those properties that is required.

For under 3. is 24. which is to greate: and under 6. there is 12. which is to greate also.

But under 8. standeth 9: which is to litle, by a greate deale.

Mester. Then proue in this other nomber. 132.
Scholar. His partes will stande thus.
The seconde parte

Where I see quickly that it can not bee a diameterall number. For the numbers under 3. and 6. be to greate: with no number that should bee sette under 3. maie be aboue 4.

Other under 6. maie any number bee set greater then 8. As it dooth sufficiently appeare by that that is taughte before.

And under 11. there can bee no lesse number placed then 60: and therefor 12. is to smalle.

And herein I perceiue greate helpe by this table, whiche you have set forthe.

Maste. It is well marked of you. But yet trie this other example. 6 0 7 2.

Scholar. I set downe his partes in order, thus.

<table>
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<tr>
<th>3.</th>
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<th>8.</th>
<th>11.</th>
<th>12.</th>
<th>22.</th>
<th>23.</th>
<th>24.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024.</td>
<td>1012.</td>
<td>759.</td>
<td>552.</td>
<td>506.</td>
<td>276.</td>
<td>264.</td>
<td>253.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>33.</th>
<th>44.</th>
<th>46.</th>
<th>66.</th>
<th>69.</th>
</tr>
</thead>
<tbody>
<tr>
<td>184.</td>
<td>138.</td>
<td>132.</td>
<td>92.</td>
<td>88.</td>
</tr>
</tbody>
</table>

And here I see a greate sorte of numbers, whiche can not serve to my purpose, because those that bee euen, and are lesse then 44. make to little a square, to be 4. times so moche as the number under any of the.

And 44. maketh to greate a square: wherefore it can be none of the euen numbers.

Again, those that be odde under 23. doe make to little a square, to bee double to the greater number under it. And those that bee odde above 23. doe make to greate a square. So that 23. doeth remain to bee the true number for the lesser side: and 264 the greater side.

Maste. Because exercise is the belse instrument
of Arithmetike.

in learning: therefore will I propounde to you one example more.

What saie you of, 5 460? Is it a diametrale number or no?

Scholar. I will trie it, by setting dowe his partes thus.

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<tr>
<th>3.</th>
<th>5.</th>
<th>6.</th>
<th>7.</th>
<th>10.</th>
<th>12.</th>
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<tr>
<th>20.</th>
<th>21.</th>
<th>28.</th>
<th>30.</th>
<th>35.</th>
<th>42.</th>
<th>52.</th>
<th>60.</th>
<th>70.</th>
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</thead>
<tbody>
<tr>
<td>273.</td>
<td>260.</td>
<td>197.</td>
<td>182.</td>
<td>156.</td>
<td>130.</td>
<td>150.</td>
<td>91.</td>
<td>78.</td>
</tr>
</tbody>
</table>

And here I se diuerse and many numbers, whiche at the firste sighte, appere nothyng mete for this purpose. For 20. is to smalle a number, as I maie some judge: and therefore all other numbers under it, must necesse be to smalle, of force.

Againe, I see that, 30. is to greate a number, and therefore, of necessitie, all other numbers aboue it, must necesse be to greate. So that 21. other 28. must be the true number, or else none.

Wherefore I examine first, 21. whose square is 441 whiche should bee one more then double, to the number under it, that is to saie, it should bee 521. And so it is not: Therefore I refuse it, and examine, 28. whose square is 784. And that should bee fower tymes so moche as 195. (whiche is the number under it) and 4. more. Therefore I doe quadrique, 195, and it maketh 780. And then I see that it wanteth, but fower of the other square: wherefore I take those twoo numbers, I mean 28. and 195. for the true sides of 5460. Whiche I finde to be a diametrale number.

Walter. By the waie, remembe that you could easilly perceiue, that all numbers under 20. were to small for your purpose: and contrary waies, all above 50.
The seconde parte

to be to great. So that you need not to sette doune so many partes of your fiste nober.

Wherefore if your nober bee soche a one, as hath many partes, you maie chose one by gesse, which you thinke will go nigh to serve your purpose: and if you finde it to finall, then set theim doune onely that bee greater then it, til you finde one other inuste: and then haue you your purpose. O2 if you finde any to great, after that which was to finall, and betwene theim none inuste, then is not your nober a diametral nober.

But and if the parte which you tooke by gesse, be to great, you shall refuse all partes aboue it, and take onely lesser partes, til you finde a inuste parte fo2 your purpose: o2 els one that is to litle.

And if in descendynge orderly, you finde no inuste parte, before you come to one that is to litle, then is your nober no diametral nober.

Scholar. This is a greate case in shortenynge of worke: whiche I will proove in this nober. 9786.

Master. If you rememberd well your former ru-

les, you would not admitt this to be examind fo2 a
diametral nober: for all the partes of the thre
peculierar termination: that is, 0.2, 0.8.

Scholar. I cõfesse my faulte. And therso2e I take

this nober. 9780. whose 20. parte is 489. But le-

yng, 20. doeth make in square but, 400. therso2e is it

very moche to litle.

Then I take the 30. parte of it, whiche is 326. and

finde it also to litle.

Thridely, I take the 40. parte of it, whiche is

244; and lesyng, 40. maketh in square. 1600. I se

that it is almoste. 7. tymes so moche as, 244; and

therso2e is it to greate.

So must the true nober be betwene, 30, and 40:

o2 els there is none at all.

Therso2e firste I take 55. whiche is the middelle

nober,
of Arithmetike.

...number (as the most apt for a conjecture) and it yieldeth 279½. And the square of 32 is 1024. Which is farre more then the double of 279½.

Therefore, again I prove with 32, which yieldeth 305½. And seeing the square of 32 is 1024, it is not 4 times so much as 305½, so that is 1222½.

Therefore I take a greater number, between it and 35. And first I take 33, which yieldeth to the 296½, whereby I may see that 33 is to greater. And seeing there is no number left between 32 and 33, therefore I judge that first number, 9780, to be no diametrical number.

Master. Examine this number, 43200.

Scholar. Because I see it to be a greater number, I will begin with a greater part of it. And therefore, I take 100, which yieldeth 432. And considering that the square of 100 is 10000, which is farre to greater, I must seek a lesser number.

Master. I will ease you of your paines in that. For because here is more to be considered. You remember that I tolde you before, in making of diametrical numbers, how that some numbers do follow the rules of others, of which they be compound. And furthermore, that such compound diametrical numbers, did bear proportion to the lesser, as the proportion was of both their sides added together.

Scholar. That is true.

Master. Of like reason all such diametrical numbers, must be excluded from these rules, which be made peculiarly for numbers that have their owne proper forms, and depend not of other.

And yet some common rule must be given, that may extend as well to them, as to any other.

Wherefore let this be it.

That the two sides of all diametrical numbers, have such a proportion together, as here you see expressed in
The seconde parte

in some one of these formes : if they bee continued as here they be begun.

The firste order.

\[
\begin{align*}
\frac{1}{4} & : \frac{7}{12} : \frac{9}{24} : \frac{4}{8} : \frac{11}{24} : \frac{8}{12} : \frac{17}{44} : \frac{15}{51} : \frac{31}{102} \\
\frac{23}{26} & : \frac{25}{31} : \frac{27}{36} : \frac{29}{41} : \frac{13}{14} : \frac{15}{54} : \frac{17}{67} : \frac{19}{90} : \text{etc.}
\end{align*}
\]

The seconde order.

\[
\begin{align*}
\frac{5}{12} & : \frac{11}{24} : \frac{13}{31} : \frac{20}{41} : \frac{24}{57} : \frac{32}{51} : \frac{36}{67} : \frac{40}{90} : \frac{44}{91} \\
\frac{48}{179} & : \frac{57}{279} : \text{etc.}
\end{align*}
\]

Here have I sette the lesser side as the numeratoz, and the greater side as the denominatoz. Whereby you maie perceiue the cause of their distinction.

For the firste order is, when the lesser side, o2 number, is odd.

The seconde order is, when that lesser side is an even number.

Stifelius doeth set them so, that the numeratoz standeth for the seconde, o2 greater side: and the denominatoz for the firste number, o2 lesser side. And for the more delectable contemplation, to behold their forme of progression, he setteth done as many whole numbers, as the fraction will give.

And this is his forme.

The firste order.

\[
\begin{align*}
1 \frac{1}{3} & : 2 \frac{1}{2} : 3 \frac{1}{3} : 4 \frac{4}{5} : 5 \frac{1}{11} : 6 \frac{5}{11} : 7 \frac{7}{11} : \text{etc.}
\end{align*}
\]

The seconde order.

\[
\begin{align*}
1 \frac{1}{6} & : 2 \frac{1}{12} : 3 \frac{1}{15} : 4 \frac{1}{24} : 5 \frac{1}{24} : 6 \frac{17}{36} : 7 \frac{11}{36} : \text{etc.}
\end{align*}
\]

Where
of Arithmetike.

Where in the first order, you see both in the whole numbers, and also in the numerators of the fraction, the naturalle order of numbers. And in the denominators, the naturalle progression of odd numbers.

But in the second order, you see that the whole numbers go in their naturalle order, and the numerators and denominators, keep an Arithmeticalle progression, by equalle distance of. 4. saue that in the numerators, all the numbers bee odd: and in the denominators, they be all even.

Now by this generall rule, if you finde any twoo partes of any number, in one of these former proportions, you make bee sure that it is a diametralle number. But foi the more apte conference of the partes, you shall doe better to reduce them to their least numbers: as you have learned in the first parte of Arithmetike.

So in your last number, which was 432 0 0, you shall finde his 180. parte, to bee 240. which being reduced to their smallest numbers, will bee, \( \frac{1}{2} \): wherefore I am assured, that it is a diametralle number.

Yet one thyng moare shall you marque.

If any number ende in Ciphers, abate euene Ciphers, as often as you can (I meane, 2, 4, 0, 6, 0, and if the reste be a diametralle number, so was the first. And therefore in this last example, 432 is a diametralle number, as well as, 432 0 0.

Also if any number being divided by any square number, doe make a diametralle number in the quotiente, then was the first number a diametralle number also.

And this, for this tyme, shall suffice for diametralle numbers.

Now will I speake somewhat briefly of like flattes: Of like and then proceede to other figurall numbers.

Scholar. I remember you defined them before, to bee suche flatte numbers, as had one forme of proportion betwene their sides.
The seconde parte

As here 27. and 12. be like flattes : because their sides be in one proportion. For as 9. is to 3. so 6 3. to 2. bothe beynge in triple proportion.

Mater. You saie well. And that is the cause why thei be called like: for the likenesse in the proportion of their sides.

Although some menne delite more to call them squarelike figures: because thei have some properties agreable with square numbers (soz as Euclide saith in his 8. booke, and 18. proposition: Every twoo numbers, beynge like flattes, have one meane number betwene theim in proportion. And the one flatte number bcareth unto the other flatte double that proportion, that their sides doe.

For declaration of whiche proposition, marke the twoo flatte numbers before: I meane 27. and 12. whose sides are in proportion Sesquialter: And the flat numbers theimselfes be as 9. to 3. that is double Sesquiquarte. Now doe you double the proportion Sesquialter, and it will make double Sesquiquarte.

Scholar. Thus doe I sette them in order. 1 : 1. And I multiplie the numeratores together, and the denominatores also. (For I remember, you tolde me before, that proportions are added, as fractions are multiplied) and then will it be. 2 : even as you said.

Mater. Again Euclide saith in the twenteth proposition of thesame booke.

If any number stande as a middle number in proportion,
proportion, between other two numbers, those two are like flattes.

That is to say: if any two numbers, being multiplied together, do make a square number (so none but soke can have a middle number between them) then are they like flattes.

As, 3, and, 12, multiplied together doe make, 36, which is a square number; and, 6, thereby appeareth to be the middell number between them. And therefore are, 3, and, 12, like flattes.

Likewaies, 3, and, 27, so they make, 81, which is a square; and their middle number is, 9.

And so are, 2, and, 8; 2, and, 18; 2, and, 50; 2, and, 72; 3, and, 48; 3, and, 75; 4, and, 9; 4, and, 16; 4, and, 25; 5, and, 20; 5, and, 45; 6, and, 24; 6, and, 54.

And so of infinite other.

This exposition is confirmed by the firste and seconde proposition of the ninth boke of Euclide, where he saith thus.

If two numbers being like flattes, bee multiplied together, the number that they make, shall be a square number.

And if two numbers being multiplied together, do make a square number, then are they like flattes.

By whiche rules it doeth appeare, that you ca have no progression Geometricalle, but it must be made either of square numbers, or else of like flattes, whereby there appeareth a greate agreeablenes, between like flattes, and square numbers. And therefore saith Euclide also in the 26. proposition of the eight booke.

Numbers that bee like flattes, have soke proportion together, as one square number bea-
The seconde parte

reth to an other.

This maie you proue by any of the former exam-
ples. Foz. 12. to. 3. is in like proportion as. 16. to. 4. 02. 36. to. 9.
Also. 27. to. 3. hath like proportion as. 36. to. 4: 02 14. to. 16. other. 81. to. 9.
And farther, if you deuide the one of theim by the
other, the quotiente will be a square nomber.

Scholar. That doeth appeare evidently at the
firste deawe.

Foz. 12. diuided by. 3. doeth make. 4. And. 75. diu-
ded by. 3. giueth. 25.
So. 5 4. by. 6. maketh. 9. And. 72. by. 2. yeldeth. 36.
And so I see in the reste, that all the quotientes will be
square nombers.

But I desire moche to knowe, how those nombers
be produced. Foz. that I knowe not yet.

Mater. Take any twoo square nombers, what
do euery thei bee, and multiply them by any one nom-
ber, that you liste: and thei will make. 2. like flattes.

So. 4. and. 9. multiplied by. 2. doe make. 8. and. 18:
whiche bee like flattes.

Again, if you multiply them by. 5. thei make. 20.
and. 45. whiche bee also like flattes.

Scholar. I am perfect enouogh in this, if that be al.

Mater. An other waie you maie make them al:
so: If you take any twoo square nombers, that will
admitte one diuido, and diuide them bothe by it.

As for example. Seyng 9. and. 36. will be bothe di-
uido by. 3. I doe so diuide theim: and thei quotientes
are. 3. and. 12. whiche are diametralle nombers.

So in like maner, if I diuide 196 and 49 (whiche
bothe are square nombers) by. 7. the quotientes will be
28. and. 7.

Again, 16. and. 100, seynge bothe square nombers
and
of Arithmetike.

and divided by 4, do make 4, and 25, as their quotient, and the be like flattes.

Scholar. And in these I see an other strange woorke: that if those two like flattes be multiplied together: thel will make the greater square, of which thel came.

For 3 tymes 12 maketh 36: and 7 tymes 28 geth. 196: And so 4 tymes 25 byngeth for the 100.

Master. It doeth so happen often tymes: but it is not alwayes so.

For if you divide 16 and 100 by 2, the quotientes will be 8 and 50, which the two numbers multiplied together, do make 400, farre differing from 100.

So 36 and 196 byng bothe the square numbers, and divided by 2, do make 18 and 98, which be like flattes: and those like flattes multiplied together, doe yeilde 1764, which is a square number, but it is 9 tymes so great as is 196.

Scholar. Yet one doubt I have: whether all square numbers be like flattes, and so be not distince from them?

For although in the division of figuralle numbers you did distince them, yet in the examples of like flattes, you put certain square numbers amongst other.

Master. All square numbers are like flattes, byng compared together: and els not. For as any 2 square numbers maie be compared together: so maie thel be referred to their roots, without comparison together. Or els thel maie be compared to other numbers that bee not square.

Therefore marke these two rules well, that no one number can bee called a like flatte: but in comparison to some other. For 2 by himself is not called a like flatte, excepte he bee compared to 8, 03 to 18, other to 32, 03, 00, 02 some other loche.

So like waies, 4, whiche by nature is a square no-
The seconde parte

ber, and awales shall bee so; yet is it not accepted as a like flatte, onles it bee referred to some other square number.

Scholar. What if it be compared with 12, which you named before to be a like flatte?

Master. You remember: one of Euclide his rules (whiche I repeated before) is, that like flattes being multiplied together, will make a square number. And sodoeth not. 12, being multiplied by 4.

Scholar. Now I doe understand your woordes better. So, 3, and 8, compared together, bee not like flattes: yet eche of them compared to other numbers, maie be like flattes. As, 3, compared to 12, 02 to 27: and 8, compared to 18, 02 to 50.

Master. Now will we lette these like flattes alone so; a tymce: And intreate moze of rooted numbers. And first I will tell you somwhat of the names and natures of soche numbers as have rootes: Then secon
darily I will teache you the order to extract their rootes: And afterwarde will I shewe some parte of the use of them.

Wherefore to begin, where we lefte a little before, the explication of rootes: I saie, that the roote of number, is a number also: and is of soche sorte, that by sondrie multiplications of it, by it selfe, 02 by the number resultyng thereof, it doeth produce that number, whose roote it is. And according to the number of times that it is multiplied, the number that resulteth thereof, taketh his name.

So that one multiplication maketh a square number. And twoo multiplications doe make a Cubike number.
Likewaies, 3. multiplications, doe give a square of squares. And 4. multiplications doe yeilde a surfolde.
And so infinitely.
For as multiplication hath no ende, so the numbers amountyng of them be innumerable, and their rootes
of Arithmetike.

rootes as infinite. But their names thei take certain
ly, of the numbers that thei doe make.

So the roote of a square number, is called a Square
root; and the roote of Cubike number, is named a Cu-
bike root; In like sorte that roote is called a Squared
square roote, whiche makeith a square of squares in nu-
mer. And that root is a Sursolede root, that yeldeth a
Sursolede number: in whiche sorte of multiplication, you
make proceed infinitely, as I said.

Notwithstanding for your ease, I have set foorth
here in a table, certain of the moste notable kindes of
rooted numbers.

And to the intente you make partly conceive the
reason of their names, I will after the table, set foorth
a briefe explication of their names, with the protractor
ure of the figures, that thei doe resemble in multi-
plications Geometricalle: where pointes, lines, platte
fozmes, oz sound佐zmes bee multiplied: and bynge
fooz the other fozmes agreable to fuche multiplica-
tions.

But first make the table well: And it will give
you greate lighte, and aptnisse to underst
ande all that foloweth, moche the
better.

For exemples are the
lighte of tea-
chynge.

The
<table>
<thead>
<tr>
<th>Cubic Cubes</th>
<th>Cubes of Cubes</th>
<th>Squares of Cubes</th>
<th>Cubes</th>
<th>Squares</th>
<th>Roots</th>
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The table of rooted numbers.

The authors.
of Arithmetike.

Here you see diverse rules of numbers, and against every rowe two names written: one on the right hande, and the other on the lefte hande, whiche serve for all the numbers in that rowe.

The names on the lefte hande bee those names, whiche bee commonly used, and attributed to those numbers.

The names on the righte hande, are names of my addition, whiche doe aptly express the very natures of the numbers, unto which thei bee assigned: as alone I will declare.

And now concerning the numbers, you see firste in the heede of the table, a rowe of numbers set in order, as thei followe in common numbering, from one sozward. And thei bee called rootes, for that the multiplication of eche of them, by theinselves, or by that, that thereof amounteth, byngeth for the all thother, that bee set under them. Of the whiche, the seconde rowe is called Square numbers: because that their length Square and their breadth (whiche I understand by the two numbers) is equal.

As 2 tymes 2 doeth make 4, whiche is a square number, and maie bee figured thus.

And in like waies 3 tymes 3 maketh 9, whiche is a square number, and is represented thus.

And here you see, that if you divide the Square number by his roote, the quotiente will be the same number also.

Scholar. That must needs be so.

Mater. Then in the third rowe are placed Cubike like numbers: whiche are produced by triple multiplication. As 2 tymes 2 twise, maketh 8. And 3 tymes 3 thirze, makebeth 27. So 4 tymes 4 fourer tymes giveeth 64. These numbers can not be expressed aptly in nattre, but prospectively, as Dice maie be made in protracture.

V.s. And
The seconde parte

And these are their fyrmes.

In the firste figure you see 2. exprested in lengthe, bredthe, and dephthe. And in the seconde foyrme, 3. is represented in all those 3. dimensions. In the 4. figure 4. is the roote, and is drawen agreeably to that foyrme.

Scholar. This is manifest enough to sighte.

Master. Yet reason ought to weigh it more exactly, then sight can comprehend it. For as their triple multiplication doth resemble the nature of sounde bodys, so it might appeare more justely expressyng of their figures, agreeably as sounde bodys ought: in whiche every parte can not appeare to sighte, with diverse of them loke inwardly. As by these 3. laste fygures you maie partly conjecture. Off whiche at this tyme and in this place, some men will thinke it an oversighte to speake, and moche more oversighte to write of them any thyng largely. Sane, that we maie sle theim for the after explication of that triple multiplication,
of Arithmetike.

...lication, wherby thei be made.

So that as it is multiplied thise, to the number
that doeth amounte thereof, hath gotten, 3. dimen-
ones, whiche properly belongeth to a bodie, or sound
forme. And therefore is it called a Cube, or Cubike nomber.
Whiche nomber if you divide by the roote, the quoti-
ent will be the square of the same roote. As I said afore.

But to procede, if you doe multiplie that Cubike
nomber by his roote, the number that riseth of it, is
called a Square of squares commonly: because that not Squares of
only it is a Square nomber, but the roote of it also is Squares.
a Square nomber. As you make perceiue by examina-
tion, of all those nombers that be in the fourth rewe,
whiche nombers I doe call Longe Cubes: because thei Longe Cubes.
make a line of Cubes. And hath in lengthe so many
Cubes, as the first roote doeth containe unitics.

This line of Cubes, although it haue for his
brede the, and depte the also, the thickness of one Cube,
yet because it hath no nomber of Cubes, in brede the,
noz in depte (o2 generally no nomber of that thyng,
whereof it is called a line) therefore maye it tollerably
beare the similitude and name of a line. And so doe
we commonly call lines, those smalle cordes, whiche
are onely long, and haue little brede the to their length.
But yet are thei not without all brede the.

Scholar. And thereof (I thinke men call a line of
Buckes, and a line of Ascheles stones, when many
beu laied in a rowe, in lengthe: and but one (o2 seue)
in brede the.

Mastuer. You saie truthe. And that name doeth
continue still, emongest all our countrie menne: save
that moste menne do not call it Sharpely a line, but
more breder (after tholde Englishe language) a laine
And so men use to saie, a laine of wine buttes, and a
laine of brede clothes: and soche other like.

And use hath so largely applied this name, that it
H.I. maie

Unsure
The seconde parte

make some no greate absurditie, to name any thynge
a line or laine, that hath moche more lengthe then
brede: and is made by often addition, or multiplication
of any one quantitie. But yet for avoiding of
erroure, it ought to bee limited, whereof that line is
named. As in our mater to sake, a line of units: a line of
diffes: a line of Cubike Cubes: and a line of Cubike Cubes Cubike: and so forthe.

In like waires must we judge of platte fomes, that
they have no depth, or thickenesse. When one nom-
ber is multiplied by an other, onely twice: that is to
saie, in brede and lengthe onely: and is not multi-
plied the third time by any number, to make it beare
depthe.

And this must be considered generally, though the
number so multiplied bee a Cube, or any other sounde
number. For in soche case, that Cube, or sounde number,
what so ever it be, standeth but as an unitie.

Scholar. Sir, I doe very well unde[r]stande the
meanyng, and reasonableness of those names, line,
and square, in any thing. But I knowe not those ter-
mes, Cubike Cubes, and Cubike Cubes Cubike. Although
I se them set in the table, whiche you have given me.

Master. No moze then doe you understande dif-
verse other names there, whiche I wil therfore des-
clare unto you.

If you agree to the use of the name, of a line and a
square, in that sorte that you have consented unto:
then if I multiply a Cubike number by his roote.
As to saie, 8. by 2. 9. by 3. other. 64. by 4. then
shall I have a line of Cubes, whiche I doe therefore
call longe Cubes: but commonly they bee called Squared
Squares, or Squares of Squares: and of some men they are
named Zenzizenzikes, as square numbers are called
Zenzikes. Whiche name although in sounde bodies,
it hath no use, yet in practice of sounde numbers, it
maie
of Arithmetike.

maie and doeth erreffe some properties aptly. As
namely that all those numbers, whiche rise of 4 mul-
tiplications, maie be as well made by twoo multipli-
catiōs. But then the roote of that multiplication shal
be a square nomber also.

Scholar. So I understande that. 16. Is a number
of that sorte, which here is called Square of squares. And
yet maie it bee called a square number: and is so in
deede, in comparison to. 4. And therefore, I perceiue,
it is set twayne in the table: ones emongest square nom-
bres, under 4 whiche then is his square roote: And a-
gain it is set emongest squares of squares, under 2 whic
in that place standeth as his squared square roote.

Likewaies. 6 4. Is twayne set in the same table, ones
emongest squares, under 8. Whiche is his square roote:
And again emongest Cubike nombers, under 4. Which
is his Cubike roote.

Master. You saie trueth. Although the laste exa-
ple be not to your purpose, concerning Squared squares
or Zenzizenzikes. And if you did note it onely, forbi-
cause it is twayne set in the table: then maie you see it
thwise sette in the same table, for it is in the tixte rece
under 2.

Scholar. So I see, wherefore I might rather have
take 8 1. Whiche is a Zenzizenzike number, and so hath
for his roote. 3: And also it is a square number, and
hath. 9. For his roote.

Master. Farther to procede, if I multyple those
squares of squares by their roote, they will make Surfo: Surfolides.
side nombers.

Scholar. I perceiue by the nombers in the table,
that you meane the leafte roote of the twoo: because
under. 16. I see. 3 2. In the rece of Surfolides.

Master. Reason maie dyme you to thinke so. For
the number and his roote, muste beare allwaies one
name. So that if I name. 16. As a square nomber, I
H. iy. must
The seconde parte

must referre it to his square roote. And if I name it as a Zenzizenzike nomber: it muste be referred to his Zenzizenzike roote. And in like sort of all other names.

As when I call 64, a square nomber, I demaunde what is his roote: you muste nedes answere by his Square roote, whiche is 8. But if I name 64, as a Cube, and doe then seke for his roote: you must understand his Cubike roote, and that is 4. But if I name it to bee a Square of Cubes, or Zenzicube: then is 2, his roote. As you maie by the table perceiue. And also by the orderly multiplication of every rewe, or order of nombers by their roote. For therby amounteth the nerte rowe.

And so maie you increase the nombers of those re- wes, or orders, according to the tymes of your multiplication, as moche as you list. And evry order shal bee afoare the names, as agreeith to the nature of their rootes.

Wherefoze thei appeare to bee oversene, that call those sozmers nombers Surdesolides, seing thei are not any waires Surde nombers, but haue their rootes. And yet, to confesse the truthe, I cannot well tell you the true etymologie of their name: except thei be so named, as it were solide upon solide. And that interpretation were to streightly raked. But the name being received and well knownen, wee maie more easily with libertie bse it, then with scrupulositie, curiously se a it.

These nombers are simple nombers in their kind. For the rise of 5, multiplications. And if their roote bee a digite nomber, then is it thesame nomber, that standeth in their firste place. And if their roote be an article, then hath that Surfolide. 5, tymes so many Cy- phers together in the firste places, as his roote hath: and the nerte figure after those Cyphers, is the firste figure significative of his roote.

Scholar. I see it so in all these nombers, that bee
of Arithmetike.

in the table.

Master. And so shall you finde it in all others.

And farther if the roote bee a number mirte, then the firste number of the surfolide, is the first number of the roote. And this I doe tell you for some helpe, in getting at their rootes.

This name therfore of theim, I meane Surfolides, in Arithmetike, maie seere to admonishe you of their roote. But in Geometrie, 92 in composition of sounde bodies, it serveth to no vse: and therfore I doe call the agreeable to their figure, Squares of Cubes: because the Squares of make a square soyme: but so that every unitie of that square, is in it selfe a Cube: As by the figures that followe, you maie well conjecture.

And also thei are made by multiplication of a Cubike number, and a Square number together, bothe ha-.
The seconda parte

Seconde surfolides. bisurfolides, oj bissurfolides, that is, seconde surfolides, oj double surfolides. But I make call them seconde squares of cubes, alluding at the same name. Howbeit if I looke to their forme and nature, I shall be inforced to call the, longe cubes of cubes, oj longe cubike cubes.

And so by like reason, doe I call the nexte numbers square cubes of cubes, oj square cubike cubes: whiche other men doe call zenzizenzizenzikles, that is squares of squared squares.

The ninth rewe of numbers is commonly called Cubike Cubes, oj Cubes of Cubes: because the Cubike rootes of those numbers are Cubike numbers also. But I after their true nature, doe call them Cubes of Cubes Cubike: oj Cubes of Cubike Cubes.

The tenth rewe of numbers is named vulgarly, squares of surfolides, because they have a Square roote, whiche is or it self a surfolide number. And so, their figure Geometricalle, I name the longe cubes of cubike cubes.

So that I considering their nature, that the one figureall numbers, am constrained to name them, according to their figure, I meane in this place, where I doe make explication of their natures and names.

But other men to aide of wooze, in extraction of rootes, have given them soche names, as maie be the best put menne in remembrance of redy wozke therein. Whiche names I will bee also hereafter, in my wytynes, because I will not bee an author of vnneede full singularitie. And yet because truth in nature is as well to be regarded, as ease in wozzyng, and rather more, I could not omitte in this place, the declaration of their true nature and very formes.

And so bothe of us hauyng good reasons, for those names, neither maie conteynpe other, neither contends together.

A generall. And although the names that I doe giue, male reason for na seme to some menne (whiche are scarce apte judges) more
of Arithmetike.

more obiouse, for the newe invention (as thei maie mes of thesethinkesthen nedesfull to the practise of tharte: yet that you see in them a naturall sequele, and orderly propagation.

For all those numbers are considered, in one of 2 soymes firste. That is to saie, other thei bee taken as numbers absolute, without any consideraion of multiplication. And so thei maie be named numbers one only, without name of relation. Or els thei bee considered as numbers multiplied, and that can be but in 3 varietes.

If thei be multiplied but ones, then doe thei make a line of numbers, or a linear number. And that nober hath onely lengthe, without bredthe, or depth: And therfore maie be the roote to a Square, or a Cube. But is of it self, in that consideraion, nother Square nor Cube.

Secondarily, it maie bee multiplied twice, the one number adyng for the length, and the other for the bredthe: and so is it a Square number, and therfore a flat number.

Thirdly, it maie bee multiplied thrice, and therby gette lengthe, bredthe, and depthe: whereby it is made a sounde number. And because the sides bee equalle, it is specially a Cube or Cubike number.

Hows can there be no fowrther waie, that any multiplication maie increase: for there are no more dimensions in nature.

But if any manne doe multiple the fowrthene tyme, then must he accoumpte that he maketh a line of Cubes: and the fifth multiplication maketh a Square, in whiche every unite is a Cube: So the firs multiplication maketh a Cube of Cubes, accoumptyng every letter Cube for an unite. And there is a fale again.

Wherefoze if any man multiplye the seuenthe tyme, he returneth againe to the firsse nature of numbers I. I. multiplied,
The seconde parte

multiplied, whiche are liniarie numbers: And the 8. multiplication, woorketh as the seconde did, and makest flatte numbers. The nineth multiplication agreeably with the thirde, doeth make Cubes.

And so infinitely these. 3. woorkes maie bee reiterate, but a souther forme can never be devised.

And therefoare doe I, as reason doeth compell me, reduce all numbers to those. 3. forms, as their verie originalle springes and fountains.

But to the intente that you maie the more aptly judge of them, and their natures, I have here sette sooth the the forms, whiche thei make in figures Geometricalle, 92 sounde quantities. Admonishing you to remember this well. That after any number is become a sounde number, it is against reason, to reduce him to an absolute flatte number again, and moste of all by multiplication. But now marke these figures.

Rootes, or Lines.

\[
\begin{array}{ccc}
4. & 3. & 2. \\
- & - & -
\end{array}
\]

\[
\begin{array}{cc}
6. & 5. \\
- & -
\end{array}
\]
of Arithmetike.

Squares.

\[
\begin{align*}
2 \times 2 & : 4 \\
3 \times 3 & : 9 \\
4 \times 4 & : 16 \\
5 \times 5 & : 25
\end{align*}
\]

Cubes.

\[
\begin{align*}
2 \times 2 \times 2 & : 8 \\
3 \times 3 \times 3 & : 27 \\
4 \times 4 \times 4 & : 64
\end{align*}
\]

Longe Cubes.

\[
\begin{align*}
2 \times 2 \times 2 \times 2 & : 16 \\
3 \times 3 \times 3 \times 3 & : 81
\end{align*}
\]
Squares of Cubes.

2^2 \cdot 2^2 \cdot 2^2

3^3 \cdot 3^3 \cdot 3^3

Cubike Cubes.

2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2

3^3 \cdot 3^3 \cdot 3^3 \cdot 3^3

Here
of Arithmetike.

Here, as you see, I have set first certain lines, containing soche partes as the bee made of by multiplication; that is to say, 2, 3, 4, 02, 5. And these bee produced by the firste multiplication, where an unitie of any thing is multiplied by a number.

And so an inch multiplied by 3, maketh 3 inches: And a foote multiplied by 6, maketh 6 foote: and so of other measures and quantities, in like sorte. All which multiplications, doe maketh only longe lines, or measures in lengthe onely, without bredth or thickeless.

And in this multiplication, neither the number, neither yet the unitie, is accounted or called a roote. But the line that is made therby: make bee a roote to any of all the other kinde of numbers before rehearsed, and sette forth in the table. For if you multiply the same line, by the number that his lengthe doeth include, then there will be made thereof, by this seconde multiplication, a square figure, containing a square number in it: As you see amongest those figures, the firste soower to be, which are marked with these numbers, 4, 9, 16, and 25.

Scholar. I perceiue well in eche of the, that their lengthe is agreeable with their bredth; and so thet make square figures; but I knowe not what those numbers doe meane, that be set over their heades.

Master. The quantitie of the number, doeth be taken the bale of their roote. And the multitude of the same number repeated, doeth declare the number of multiplications, for eche figure.

And therefore the lines, which are made by one multiplication, have eche of them their number simply set, ones onely.

The squares have their numbers double: in token that thet haue 2 multiplicactions. That is, one in length, and another in bredth.
The seconde parte

The third fowmes, whiche be Cubes, and are made of 3. multiplications, haue their rootes repeated thricis.

And the like numbers did I sette, in the side of the former table, against the like quantities, Whiche shal helpe you somewhat in the extraction of rootes.

Scholar. How doe I perceiue not onely their names, and multiplications, moche better then I did before: but also I understande better the difference of your names, and their reasons. F02 by those figures, whiche you haue set in the wowerth place, and doe call theim longe Cubes, I see their fowme doth agree to that name. F02 thei are longer, then thei are other br0de or depe. And saue for their depe, I might liken theim to longe Squares in Geometrie. Howbeit, othere men neglectyng their fowme, and looking onely to their rootes, doe call them Squared Squares.

But if you will permitt me, to speake in the defendede of theim, as a simple schollar male speake so affection, in the defence of his master, it appereth to me, that thei male well bee called Squared Squares; and might be figured thus.

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2.2.2.2.          3.3.3.3.
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Where the smallest squares, whiche be contained within the pricked lines, beynge taken as rootes, and multiplied
of Arithmetike.

multiplied by the same number again, which they do contain (other els twice by their rootes) will make the whole greater squares.

And by this figuring of thim, ther doctr appere no inconuenience nor absurditie, in their vulgar names: but rather a taste expressyng of their naturalle forms.

For in the first figure. 2. Standing as the side of the lesser square, and multiplied by it self, doeth make 4. which is the quantite of the lesser square. Then if I multiply that lesser square, 4. by his owne number, it maketh 16. which is the greater and whole square: and is a Square of squares.

So in the second figure. 3. Standyng for the roote of the lesser square, contained within the pricked lines, and if it bee multiplied by it self, it maketh 9. which is the quantite of the same lesser square. Then if I multiplye that, 9. by it self, it will make 81. which is the quantite of the great Square, and is a Square of squares.

Master. I commend ye you well: not onely for so diligent searchyng of thim, which for their honeste travell, deserveth moche thankes, but also for that ye seke to bring manifest reason, and some shewe, at the least, of linearie demonstration for your purpose. So that you will not seme to speake, without some good grounde.

But as in deede, your figure doeth truely express a square of squares, so it doeth suppose the other number, which by order of multiplication, doeth go next before it, to be a flatte number also. For it is not possible that a sounde number (as a Cube is alwaies) being multiplied by any other number, maie leave the nature of a sounde number: But shall continue a sounde number still. And therefore seyng the nexte number, before a Square of squares was a Cube, it is not possible
The seconde parte

possible that a Square of squares can be a mere slatte nombre, as you have drawn it.

Whereof if they had intended, that a slatte nombre should occupie the .4. place, then should they have set some plate forms in the third place also. Whiche might have been made in this sorte.

And then will it be a longe Square, and not a Cube.

\[ 3 \times 3 \times 3 \]

But in as moche as they doe not admitt this longe Square (whiche by that name hath no roote) therefore make not the nombre that foloweth it, bee any other then a founde nombre. For every Cubike forme, being multiplied by his roote, doeth make a Square piller. Whose length beareth unto his breeth the same proportion, that his roote doeth unto an unitie.

Scholar. I am very well satisfied now concerning the names and formes of those numbers. And by this that you have saied, I doe further perceive, that 5. multiplications doeth make the square of Cubes, whiche be set in the fiftie place, emongeste the former figures. And also I understande by the former table, that thei be called Surfordes.

Likewaies I see in the fiftie place of the soyled figures, Cubike Cubes, made by 6. multiplications. But commonly the numbers of those quantities, be named Squares of Cubes. So that for their names, thus farre I am perfecte enough.

The
The extraction
of Rootes.

Walter.

Owe will I chewe you, the extract how you shall extract the root of rootes out of any soche number.

And first I must admonishe you, that you shall alwaies understande, soche a roote, as the number dooth admit. So that in a square number, you shall seke a square roote onely, and no Cubike roote, nother any other kind.

Likewaies a Cubike number hath no other rootes, but a Cubike roote. Excepte the name bee compund, as Zenzicubike, or Squared Cube. Fō there there are 2 rootes of rootes, according to the 2 names that the beare. That is bothe Square and Cubike roote: as I will anon chewe you. But first I will beginne with Square numbers, and their rootes.

And this generall order must you observe, before all other: That you shall haue by harte, in readie memo- ric all soche numbers, whose rootes are digitae. Fō as it is superfluous to seke rules for their, so must the helpe in all greater numbers, whose rootes are above 9. And so you scarce in remembrance, I have here sette soo: the a table for square numbers.

Where in the firste colume, you see the rootes set, and in the seconde pillar, right against ech roote, there is set his square. Touching which I neede to saie no more, but that you be not in any uncertaintie of them, whi
The extraction

you shall note their aid, which shall be continually in use of searching for other greater roots.

Now for greater numbers, this is the order.

1. First set done the number as it is. Then set a pricke under every odd place, I mean the first, the third, the fifth, the seventh, and so forth; and so shall every pricke have 2. numbers, except the last, which some times hath but one.

2. Secondly, marke the numbers that belong unto the laste pricke, toward the left hande; and whether he have belonging to it one number, or two, looke what the roote maie be of that number, if it bee square. And that roote sette by a crooked line, as you place the quotiente in diuision; and cancell all that square number, belonging to that pricke.

3. But and if the number belonging to that pricke, bee not a square number, then take the roote of the greateste square, which is contained in it, and place the roote as I said before. And the square of it shall you abate from the number, that belongeth to that laste pricke, and let the rest be set over those numbers cancelled, as you doe in diuision. And so have you ended your worke for that pricke.

Scholar. This moche is caste enough, if I understand you rightly.

Mestre. Then prove it in a number, or two. And first worke with this number. 5152900.

Scholar. I must marke every odd place with a pricke, thus.

And here I perceive that unto the first 5152900, there belongeth 2 Cyphers one by, and to eache of the other 2. prickes following, there are appointed 2. figures. But the fourth pricke hath but one number, and that is 5.

Now according to the second rule, I seke the roote of 5. (soz because there belongeth no more numbers to that
The extraction

that picke) and I see, it is no square number. Wherefore according to the thirde rule, I take the greatest square in it, which is 4. and the roote of 4. is 2. Therefore I doo substracte 4. out of 5. 1 and cancell that. 5. and the 1. that re: $152900(2$ maineth, I set our 5. as here you see.

And the roote. 2. I sette behinde the quotiente line, as you taught me, and then the nobers stand, as you fe.

Master. You have done wel. Proce again in this number. 18766224.

Scholar. First I set theim doun. 18766224. and picke theim, as here doeth appeare. And now I see, that the laske picke hath two nobers belonging to it, that is 18. with whiche I must begin. And saying it is no square number, I find 16. to be the greatest square in it: wherefore I substract 16. out of 18. and set. 2. over the 8. 2

And the roote of 16. which is 4. 18766224(4. I sette behinde the quotiente line, as here is seen.

Master. This make sufficie for the first woopke.

How to procede, you shall double your roote, and put that double under the nexte space, toward the right hand, that is behinde the nexte picke. Alwaies forseyng, that if the double doe contain more figures then one, that the first shall be sette under that place, and the seconde under the nexte figure, toward the lest hande.

Then take a quotiente, as you doe in division, whiche shall shewe how often that double nober maye be found in that, that is over it, appertaining to that place: Whiche quotiente, you shall set befoxe the firste roote, within the quotiente line.

But this regarde muste you have here specially, that you make leave over the nexte picke, toward the right hande, as moche as the square of that quotiente,
The extraction

with which you worke, for out of that rest, the square of that quotiente must bee abated. And then make bothe subtractions, and note the remainder, if any be, and place your quotient, and then haue you done with that prickke also.

For the more plaines, I will give you an example in your firste number, whiche goode thus, after your worke was ended.

Here I see over the laste prickke saue one, 115, under the middell figure of whiche I must set the double of the former roote, 2, that is, 4. And then I seke how often, 4, is to bee founde in, 11. And I finde that I maie haue it twoo tymes, and 3, remaining. Whiche, 3, with 5, over the nexte prickke, doe make, 35, and that is more then the square of my quotiente, 2. Therfore am I bolde to sette downe that quotiente: And accordyng to it, to abate twice, 4. (whiche is 8,) out of 11, and there resteth, 3. Therfore I cancell, 11, and sette, 3, over it. Then doe I multiply the laste quotiente squarely: and it maketh, 4, whiche 4, I subtrahce out of the number over the prickke, that is, 35, whiche is sufficient for this number. Therfore I abate, 4, out of, 5, and cancell that, 5, and set, 1. Whiche remaineth, over the, 5.

And then will the whole number stande thus.

This worke, whiche I haue wrought now, must be repeated as often as there bee any prickes, or pricked numbers remaining. Wherby you maie easily gesse, that it must bee twice more repeated in this example, because there resteth yet, 2, prickes untouchd.

Scholar. Although I thinke, I could doe, as I haue marked you to doe, yet for more certaintie I prate
of Rootes.

praise you worke out this example.
  Parter. Then marke it well.

I shall begin againe with doublyng of all, that is within the quotiente line. And that double is 44, whiche I must set under 312, that remaineth of the last worke. And then will the numbers stande, as here you see.

| 1 3 1 |
| 5 1 5 2 9 0 0 |

Then I look how often tymes I maie finde. 44, in 312. And I see it will be abated 7 times, and 4 remain: whiche 4 with the 9, over the next picke doth make. 49. And that will suffice to extrance the square of my quotiente.

7. For 7 tymes 7, maketh in all 49. Thus seying I maie take 7, for my quotiente, I worke with it, as the rule teacheth: abating first 7 tymes, 44, (that is, 308) out of 312, and there resteth, 4, over the space before the next picke. Whiche 4, with 9, over the picke doe make 49, out of whiche I abate the square of my quotiente, 7, (that is, 49,) and so resteth nothing, but 2 cyphers. And the number standeth thus.

And seying there remaineth one picke untouched, I should repeate the same order of worke agayne, by doublyng all the quotiente, whiche would be, 454, and setting it so that, 4, whiche is in the firste place, should be sette under the Cypher, that is without the picke, and the other figures in order, toward the left hand. But all this worke were in vaine, seying there is nothing lefte, to serve for the subtraction.

Yet because there is lefte one picked place untouched, I must set for it a Cypher in the quotiente.

For this rule is generall: that how many pickes so ever your square number doth containe, your quotiente, or root shall have so many numbers. Wherfore this roote must be made by thus. 2270.

K.19. And
The extraction

The profe.

And so it appeareth that your number, 512900, is a unlike square number. Which ye maie proove by the orderly profe of extracion of rootes. That is to multiplye that quotiente, or roote (which ye have found) by it self. And if it doe make the first number exactly, then haue you wrought well.

Scholar. That profe is as certaine, as can be. And therfore I will proue, whether it will agree with this worke. Wherefore multiplnyng 2270 by it self, I see that it yeldeth the firste somme, As her I dooth appeare. So is this worke approv'd good.

And now will I attempte the like worke in the seconde example. Whiche was, 18766224.

But after the firste worke was ended, and the greatest square subtracted out of 18, it did remain in this forme. 18766224 (4).

Now to continue the worke as you did, and as the rule doeth teache, I must doule, 4. which is the roote, and standeth by the quotiente line: and must set it under. 7. that standeth in the space, betwene the laste pricke (whiche worke is ended) and the nexte pricke towarde the right hande. And then will it stande thus as you see.

That done, I must seke a quotiente, that maie declare how often 8. maie bee subtracted out of. 24. and that quotiente I finde to be. 3; because that after I haue taken 3. tymes 8. (that is, 24. out of 27. there will remain 3. whiche 3. with 6. that standeth over the pricke, doe make 36. And I see that number to bee great enough, for the abatemen of the square of my quotiente; whiche is but 3. tymes. 3. that is, 9.

Wherefore
Wherefore I sette downe, 3.
for my quotiente, before. 4. in
the quotiente line. And multi-
plying 8. by that 3. there riseth
24. which I doe subtract out
of 27. that is over. 8. and there will remain. 3. That
3. with 6. over the pukke. make. 36. out of whiche
I must abate. 9. which is the square of my quotient. 3.
and so will there rete. 27. over that pukke.

And thus have I ended. 2. pukkes, and yet. 2. more
do remain; in whiche bothe I must repeate the same
some of wooske.

Therefore I double the whole quotiente, and it ma-
keth. 86: which I let under. 276.

And then I seke the quotiente, declaring how many
tymes. 86. maie be abated out of. 276. which maie
be. 3. tymes. And so. that cause I set. 3. in the quotiente
before the. 43.

Then doe I firste multi-
plye. 86. by that. 3. sayynge. 3.
tymes. 9. make. 24. which
I abate out of 27. and there
resteth. 3. And again I sete,
3. tymes. 6. is. 18. which I
abate out of. 36. and there doeth remain. 18.

What dooen, I take the square of my quotiente, that
is. 9. which I doe subtract out of. 12. (for the. 2. over
the pukke must borowe. 1. of. 8.) and then will there
remain over that pukke. 173.

And thus is that pukke ended.

Now, for the laste pukke in wooske, though he be
firste in place. The double of my quotiente is. 866.
which I musste sette under
1732. As here is done, where I leave out many cancelled
figures, as superfluouse in

\[
\begin{array}{c}
237 \\
276 \\
187624 (433-866) \\
\end{array}
\]

this
The extraction

this place.

And then secking for a newe quotiente, I finde it to be 2, which I set with the other numbers in the quoc- tente. And by it I multiplye and subtract the 866, la-
yng 2, tythes, 8, is. 16, which I abate out of 17, and there reseeth 1. Again, 2, ti-
nes 6 is. 12 that I subtract out of 13, and there remain-
eth 1. Thirdly, I sale 2. times, 6, giueth 12, which I abate from 12, and there is left nothynge. Save that over the pricke there stand-eth 4 which is equall with the square of my quotiente.

Wherefore abating the square of my quotiente out of it, there reseeth nothynge at all.

And therby I see that 18766224 is a juste square number. And his roote is 4332.

The profe. Master. Although I knowe it to bee so, yet for your better exercise, and full perswaision: I would have you trie it, by square multiplication.

Scholar. That maie I done doe. And so I finde it to be true.

For 4332 multiplied by itself, doeth make, 18766224. As this woorie here set, doeth the we.

Master. Yet because some other small doubts, maie happen in wor-
king, that maie trouble a yong practi-
ter, I will propounde to you one or two examples more. Wherein you shall finde some varietie, as well in the number propounde, as also in the quotiente.

And firste to begin, I will you to extract the roote of this number, 22071204.

Scholar. I must set downe the number, and note it with pricke in every odd place: For that rule I perceiue
The extraction

perceun neuer falleth.

Muster. No more dooeth any of the other, although the woorke maie variue in some smalfe points: whiche yet maie be greate enouh to trouble a young learner.

Scholar. Then accorpyng to the firste rule, I seke out the greatest square in. 22. (so I see it is no square number it self) and it appereth to be 16. And his roote 4. wherefore I doe sette doune. 4. in the quotiente, and then I doe abate. 16. out of. 22. and the remainder is. 6. whiche I sette over the pricke, and cancell the. 22. as here is seen.

Now going on with the nexte pricke, I shall dou-ble the former roote in the quotiente, and sette it under the Cypher, between the 2. prickes.

Then do I seke how oft 6. (whiche is the dou-
ble of the quotiente) maie be found in 60 and I finde it to be 7 times, and 4 remainynge to be set over the Cy-
pher. So that for the pricke there remaineth. 47. out of whiche I should abate the square of my quotient. But seing that. 49. (whiche is the square of 7) can not be tak-
ken out of. 47. there is a newe quotiente to be sough-
then for. I take 6. And see that it will serue. So I let. 6. in the quotiente: and by it I multiplye 8. whereof commeth 48.

That. 48. abated out of. 60. leave-eth. 12. Therefore I cancell the. 60. and let. 12. over it.

Then doe I multiplye the quotiente. 6. by it selfe: whereof riseth. 36. And that abated out of. 127. leave-
eth. 91. And so have I ended the seconde woorke.

Now for the thirde woorke, I double. 46. and it dooth yelde. 92. to be sette under. 911. as I have put it here.

And then seking for a quotient: I se that I maie take
The extraction

9. Wherfore I set that 9 in the quotiente with, 46, and by it I multiply 92 and subtract that, that riseth, in this forme.

Nine tymes, 9, maketh, 81. 22071204 (4698)
whiche I abate out of, 91. and there resteth, 10. Then 9 tymes, 2, giueth, 18, whiche I must abate out of, 10. and there will remain, 83. And now muste I multiply that laste quotiente, 9, squarely, whereby will amounte, 81, that shall I subtract out of, 832, and there will remain, 751. and so that pricke with his woork is ended.

Therefore proceeding to the fourthe pricke, I double all the quotiente, which will be 938. And I set it under 7510. 751
Then doe I take a newe quotiente, which I finde to bee, 8. For 8, times, 9, giueth, 72, whiche I abate out of, 75. and there remaineth, 3. Again, 8, tymes, 3, is, 24. and that I deducte out of, 31. and so resteth, 7. Then saie I, 8, tymes, 8, is, 64. whiche being subtracted from, 70, doeth leave, 6. And that, 6, with the 4, over the pricke maketh, 64. out of whiche I muste withdrewe the square of, 8. that is my quotient, and it being 64, there resteth nothing. And the whole woork standeth thus:

\[ \frac{37}{7510} \]
\[ 22071204 (4698) \]
\[ 938 \]

Wherefore I sate that the first nöber 22071204, is a square nöber: and hath for his roote, 4698. As I made prove also, by square multiplicatioun. For, as in this example you see: 4698, multiplied by it, self, doeth byng for the, 22071204.
Master. Yet one example more shall you prove: and that is this. 901740841.

Scholar. I set it done, and prickle it according to the rule: And then I see over the laste prickle, one oenly number, that is, 9, which hath 3, for his square roote. That 3, I set within the quotiente line, and therefore I cancell 9.

After this I should proceede with doublinge the roote, 3, and that double should I set in the next space, over which remaineth no number, for 9, being cancelled, the Cyphor is nothing. And so am I at a stale.

Master. Seing that you can not set the double of your quotiente done there, where no number is (or if it so chance, as some times it doth, that the number over it, is lesser then the double) then set a Cyphor in the quotiente, and so have you done with that prickle.
For in soche case there needeth no multiplication, no subtraction.

Scholar. Then am I instructed fully for that pointe: The worke is so easie, I must therefore set my numbers thus.

Master. And doe you not see, that the double of the quotiente, is greater then the number over it?

Scholar. I was so mindfull of the one halfe of the rule, that I forgatte the other halfe.

But now I see, I must set an other Cyphor yet in the quotiente. And then shall I set the double of all that, in the thirde space, after this forte,

And nowe proceedynge to searche for anewe quotiente, I see that 2, shall serve me.

Wherefore I sette 2, in the quotiente line, with 300. And by it shall I multiply the double aforesaid: sayling 2, tymes 6, maketh 12.

L.I. to
The extraction

to bee abated out of 17. and the remaine will bee 5.

Then shall I overpaue the twanne Cyphers, because they make nothing by multiplication: and so compassing to the prickke, I hate the square of my quotient, which is 4 out of 8. and there resteth 4. Therefore I cancell. 8. and set downe 4. and so have I ended that prickke. And have but one wrope more behinde.

Therefore I set downe the numbers with the double of all the quotient, thus.

And then I loke for a new quotient, which I finde to be 9. By it therefore I multiply, first 6 and it makest 54. That doeth abate the 54. over it. Then omit I the 2 Cyphers, and multiply 4. by 9 whereby there commeth 36. Which I abate out of 44. beyng over it, and there remaineth 8. That 8. with 1. over the prickke makest 81. out of which I must abate the square of 9. beyng also 81. And so is nothing lefte, whereby it appeareth, that 901740841 is a square number, and his roote is 30029. The proofe of it doeth conforme the same. For 30029 multiplied by itself, doeth brynge to the 901740841.

The nighest roote of all square numbers.

Walter. This shall suffice for locke numbers as bee fully square. Other numbers there bee infinite, which be not square, and therefore have the no square rootes. Yet of ten tymes it happeneth, that we shall bee occasioned to search for the nigheste number, that maie resemble their rootes.

Therefore in locke case, this shall you doe. Firste extracte
of Rootes.

extract the roote, as if it wer a square number. And that roote will serve for the greatest square, that is in your former number; and there will be a remainder beside. Of whiche remainder with the quotient, you shall make a fraction, in this sorte.

Set the remainder over the line, for the numerator, and the double of the roote (that you have found) set under the line, for the denominator. And this shall be a sufficient precise number in great numbers, for any common work.

Scholar. I will by an example, taken by chance, prove this rule. For it seemeth to have no difficulty. Wherefore I take 296882.

And this, I am assured, can be no square number. For, I remember you told me before, that no such number might be a square, which had 2 for its first figure.

Then to search his highest roote, I place it, and picketh it thus.

And under 29, I finde the greatest roote to be 5, which I set in the quotient line, and cancell 29 setting 4 over it. After that I double it, and there cometh 104, that double I set in the next space under 46. Then finde I a newe quotient, which is 4 and by it I multiply 10, whereof amounteth 40, to be abated out of 46. And so remaineth 6. Again I multiply 4, by itself squarely, and there riseth 16, which I abate from 18, (saying, 8 is to small) and the remainder will be 2. So standeth the whole number, as you see. Wherefore I double the quotient, which is 54. And it yieldeth, 108, that must be set under 528 as I have here done.

Then I looke for a quotient, how often I may abate. 108, out of 528. And I see it will be but 4.

Lii. typcs
The extraction

tymes. Wherfore I set 4. in the quotiente, with the other numbers, and then doe I wooke with it: Firste multiplying 4. and 1. together, whereof commeth but 4. whiche I abate out of 5. And there remaineth 1.

Again I multiplic 4 by 4. whereof commeth 32. that doe I subtract out of 128. and there will remain 96. Then shall I take the square of my quotiente 4. whiche is 16. And that must I abate out of 962. And so remaineth 946. of whiche number set as the numerator, with the double of the roote, set for the denominator, I shall make a fraction in this sorte.

\[ \frac{94}{16} = \frac{19}{3} \]

Master. You have doen well. And so you perceiue that the nighte roote of your former number is 5 4 4 4. For these fractions are all one.

And hereby also you maye understande, that if the remainder ouer your number bee euene, you may take halfe of it for the numerator, and the whole quotiente for the denominator.

So maie you take the quarter of the remainder (is it will to bee parted) for the numerator, and the halfe of the roote for the denominator.

And in like maner generally, if the remainder and the roote in the quotiente, bee numbers communicante, divide them so, that the divisio of the remainder, bee euene double to the divisio of the quotiente roote. And so maie you easely reduce that fraction, to his least termes.

But now for prooffe of this woork, there be twoo wates: the one is certein, and the other but in nere- nest. For as the roote of soche numbers, is not a precise roote: So if you multiplice that roote by it self, it will make a number, very nighte to that sozmer number, but not exactely the same.

Whiche faulte some men thinke to reduzze, by ad-

The firste prooffe.
of Rootes.

dying of 1. to the denominator: and yet that amendement sometimes increaseth the erronie.

But because you shall not wante a sure proosse, doe thus: Multiple the quotient, or Roote of whole nom: The second numbers by it self, and unto the number that amounteth proosse. thereof, add the whole remainer. And if then it make your firste number, your woorkke was well done: els have you misset.

Scholar. That maie I prooue here quickly. The quotient in whole numbers was, 544, which being multiplied squarely, doeth yeeld 295936. unto which number, if I doe adde 946. that did remaine, it will amounte to 296882. and that was the number proponed to me: wherfore it appereth that the woorkke was well done.

Master. You shal neade no more examples, for this forme of woorkke.

But one other waie wil I shew you, how you shall gette verie nigh unto the root. And another you shal go as nigh as you will desire, in any place to finde like woorkke. If you desire to gette within leste then 10. the nighest of one, then set before your number. 2. Cyphers. And roote. If you would not erre, then set downe 4. Cyphers: But and if you like to sette downe 6. Cyphers before your number, you shall not misse of an unitie fro the true root. And if you like to go any higher in preciseness of partes, addde still euene Cyphers.

Scholar. I would faine prooue this forme, in the same example, whiche I wroughte laete: Because I would se the agremente betwene the bothe woorkkes.

Master. Go to your consideracion is reasonable And because the partes maie the better agree, sette downe 6. Cyphers. And then shall your roote exeell thousande partes of the whole number.

Scholar. I sette downe the number, and pricke it thus,
The extraction

thus. Whereby I perceive that I shall have the same order of woorke, and the selfe same numbers that I had before, till that I come to the Cyphers and their pickes.

Master. True the it is. And therefor maye you in soche a case sette doune onely the remainder, with the Cyphers. Or els cancell all the numbers, saue the remainder, and the Cyphers: and set the former whole roote, without the fraction, in the quotiente.

Scholar. Then will
it stande thus.

Now accooyng to the rule I will proceade: as this whole nober wer the first number proposed unto me. And therefor I doe double at the quotiente, whiche makes, 1,088. and that doe I set under, 9,460. And then shall I seke a quotiente, that may declare how often tymes, that double is contain in the number over it. And I sett it will bee, 8. Wherefore I set doune, 8, in the quotiente, and by it I multyplie the double, and subtraxte it, in this sorte: saying 8 tymes 1, out of 9, leaue, 1, remained. Again, 8 tymes, 8, (that is, 64,) out of 146 will leaue, 82. Then farther I abate, 8 tymes, 8, out of, 820, and there reft eth, 756. And last of all, I take the square of the quotiente, which is also, 64, out of 7560, and there will remain, 7496. And so have I done with the firste picke of the Cyphers.

Master. Consider now that by those 2 Cyphers you have gotten 8 into the quotient more then you had before. And all your former number of the roote, removed by it into one place higher, then it was before.
of Rootes.

So that, where by the first worke, your roote was 544, and almoste 2 : by this worke you have founde it to bee \( \frac{5545}{544} \), and \( \frac{5545}{544} \) of \( \frac{1}{11} \) : which is verie nigh the same number, that you had before.

Scholar. In deed, if I reduce the fractions, it will bee 544, \( \frac{8}{11} \) and \( \frac{912}{112} \) of \( \frac{1}{11} \) : which is in one fraction, \( \frac{1143}{112} \) above, 544.

Master. Marke this triall. And use the like after every twoo Cyphers are ended : And you shall see a goodly agremente of the woorkes together.

Scholar. In the meane tyme, to procede with the former woork, I set done the number with the remainder, and the doble of the quotiente, as here appeareth.

And searchyng for a newe quotiente, I finde that it will be 6.

Therefore I sette done, 6, in the quotiente with the other numbers. And by that 6, I doe multiplye the double of the whole quotiente, and subtracte it orderly, saiyng 6. times. 1. being abated out of 7. length. 1.

Likewaies, 6. times. 8. makes. 48, whiche I shall abate out of 49. and so resteth. 1. Then 6. times. 9. (whiche is 54.) must be subtracted out of 1016, and there will remaine 962. Againe I shall abate 6. tymes. 6. (that is 36.) out of 9620, and there is left. 9584. Then take I the square of my quotiente, whiche is also 6 times 6, or 36, and that I must abate out of 40, and there resteth. 4. And thus is the seconde pricke of the Cyphers ended.

And now I finde in the quotiente not \( \frac{1}{10} \) as I did in P. 1. the
The extraction
the laste woorkye before this. But I finde \( \frac{5}{6} \) whiche goeth more nighte to \( \frac{3}{2} \). For \( \frac{9}{10} \) would be \( \frac{5}{6} \) and \( \frac{5}{6} \) is equale with \( \frac{5}{6} \). And I make easily se, that \( \frac{8}{10} \) is more nighter to \( \frac{9}{10} \) then to \( \frac{5}{6} \) beside the remainer, whiche will make \( \frac{4}{10} \) of \( \frac{1}{4} \), or els \( \frac{4}{10} \) of one.

Paster. I se, a well willyng mynde can marke diligently, and learn speddily; wherfore go so forwarde with your woorkye.

Scholar. I muske sette doone the double of all my quotiente, whiche will be 108972. And it will stande thus.

\[
\text{Wherefore I doe take for a newe quotiente, and I finde it to be} \frac{95802}{29688209000000} \text{of } 108972.
\]

Paster. And the other numbers, and by it I woorkye after my rule, saiyng 8 tyme 1 is 8, whiche I abate from 9 and there resteth 1. Then take 1 8 tymes 8 (that is 64) out of 158 and the remainer will bee 94.

Scholar. Again I subtract 8 tymes 9 (beiyng 72) from 940 and there is left 808. Farthermore I take 8 tymes 7 (whiche is 56) out of 82 and there restoth 26. Then doe I withdrawe 8 tymes 2 0z. 16 out of 60. And 29688209000000 (544868) there remaineth 44.

Paster. And now I have for the roote \( \frac{544}{2} \) that is \( \frac{544}{2} \) and beside \( \frac{1}{4} \) of \( \frac{1}{2} \) in lesser termes \( \frac{107797}{133217} \) of \( \frac{1}{2} \), whiche beyng reduced into one fraction with the \( \frac{868}{1000} \) will make \( \frac{118344153}{1332170000} \).

Paster. You have done well.

And
of Rootes.

And here you see, that you drawe nigher nigher nigher still, to the very roote, if it might have any. For if you drawe a nigher nigher number to \( \frac{2}{10} \), then is \( \frac{2}{10} \) as that was nigher then. \\

And if you would worke with more Cyphers, you should perconce still, that it would drawe nigher and nigher. But this make suffice for examples sake.

Scholar. Then I praze you tell me, what is the chief use of this rule: and for what matters it serveth.

Matter. One yere will not suffice, to express the commodities of it. It serveth to many wares, in building: in projection of plates, soz measuring of gredid Timber, or stone; And also in warfare, soz framing of battailes, soz making of diverse engines, and generally for all woorke of Geometrie and Astronomic. But for to satisfie you partly, I will sette for the twoo or three questions, that depende of this woorke of extraction of square rootes.

And firste of a battaile: because it seemeth to serve leaste for that purpose.

A capitaine generall haung three greate armyes, would caste them into three square battailes, but he knoweth not how many men, he shall set in the fronte of eche battaile.

The numbers of the three armyes, are for the firste 5625; for the second 9216; and for the third 15129

Scholar. I dooe perceiue easely, that for eche of these numbers, I muste searche out the square roote, and then have I the fronte, or farte. Sith bothe are equalle in a square battaile.

Wheresoe I set dounye the firste number thus, with his prickes. And then under the firste prake to warde the lefte hande, I finde the graste roote to bee. 7. seeping the graste square is. 49. That roote doe I set within the quoti; ente line; and his square doe I abate from. 56, and so P.s. remaineth
The extraction

remaineth. 7.

Then doe I double that roote, and sette the double
under. 7 2, and see that the newe quotient
will bee. 5. And there will remaine. 2 5.
whiche is the iust square of the last quo-
tiente.

Wherby it is evidente, that his first armie con-
tained a square number, and the roote, or side of it is 7 5.
And so many menne shall be in the fronte of the firste
battaille, and as many in the flank.

Now for the seconde battaille, I take the square of
9 2 1 6, and finde it to bee. 9 6. As in
this example I have wrought it.

For the firste number is 9. sayng it
is the greateste square roote, that can
bee founde in. 9 2. And so is the double
of it. 1 8. and the quotiente for it. 6. as it appeareth ma-
nifestly enough.

Wherfore I saie that the seconde battaille shal have
in every ranke. 9 6. men.

And now for the thirde battaille, I sette downe the
number, according to this rule: and I finde the firste
roote to be. 1. because. 1. tymes. 1. ma-
kethe. 1. And his double is. 2. whiche
I abate twise from the number over
it: and after double those bothe num-
bere, which make. 2 4. And finde
that to be abated. 3. tymes.

And so haue I gathered that the number is square
and the roote. 1 2. 3. According to whiche number, that
thirde battaille must be marshalled.

After. Sayng you are so redy in this pointe so
lore. Tell me how many menne, shal be sette in the
fronte, if all these, 3. armyes be joine into one square
battaille.

Scholar. Firste I must adde all. 3. number togeth-
of Rootes.

ther. And so will thei make 29960, as here by example doeth appere.

But this number can be no square number, because it hath one odd Cypher in the first place: for I remember your saying, that square numbers can not begin with odd Cyphers. Wherefore this number will not make a square battaile.

Yet will I prove what make be the root of the greatest square battaile, that make be made of that number.

And for that purpose I pick the numbers, and finde the greatest root 2, to be 1 and the same number to be the square also. Then double I that root, and place his double under 9, that is unpicked: and searching for a quotiente, I finde it to be 7. with whiche I woozke by the rule, and so doeth remaine fo the nexte picke. 10.

Then doe I doublle that. 17. whereby commeth 34 whiche I set under 106. And soo it I finde. 3. to be the metest quotiente: with whiche if I woozke accordingly, there will remaine. 31. as the excelle above the greateste square.

Whereby it appeareth that 29929 is a square number: and hath. 173, for his root. And that should bee the fronte of this greatest battaile.

Master. Now will I prove you with an other question of like sorte.

A Prince hath an armie verie greate. With whi: The seconde che he passeth in a Wallie, so that in marchinge the question of fronte can be but. 18. menne. And by that means the an armie stande containeth. 449352.

After that the armie is passeth that valie, the kyng mindyng to occupie all the beste grounde, will eth the battaile to be set square. How would you doe it?

Scholar, first I multiply the slanke, by the front. 

And
And so I finde the whole number to be. 8088336.

That number doe I picke as my rule teacheth me, and I finde the first roote to be. 2.
and his square. 4, which first I subtracte out of. 8, and so re-
Keth. 4. Then doe I double that quotiente, and finde that double. 8. tymes in the somme over it.

And so doe I proceede till I have founde out all the 4. figures, according to the. 4. pickes under that no-
ber. And then the roote appeareth to be. 2844.

Master. Yet one question more, to to exercice your penne, will I propounde of a like mater.

A generall hath three armes, to the number of 28289. men: and none of those three armes is apte to make a square battaile, yet he is appointed by his soueraigne, to sette them in three square battailes.

These be the. 3. numbers of the. 3. armes. In the firste there are. 10296. men: In the seconde. 9493: and in the third. 8500. Now let me see how you can cast them into three square battailes.

Scholar. I thynke it reasonable, to take the greateste squares of the first and second numbers, and the excesse of them bothe, to put to the thirde number.

Master. So are you not sure that the third number, will be a true square.

Scholar. Then knowe I not how to doe it.

Master. Take the greateste square in the thirde number also. And note those three excesses, and their rootes also.

Then put one to every roote, and marke the squares that will rise of them.

Thirdly, subtract the firste 3. numbers, out of those 3. newe squares, and note the difference of eche of the firste numbers, from those squares: and so have you. 3 numbers
of Rootes.

numbers of exceed and 3. other of wante.

Now compare those exceedes and wantes well together: and you shall easily see from whence you shall take any number, and to whence you shall add any.

Scholar. In the firste number the greatest square is 10201, and thereby the exceede is 95, and the roote 101.

In the second number the greatest square is 9409 and his roote 97. So is the exceede 84.

And in the third number, the greatest square is 8464; and the roote of it, 92. Wherefore the exceede appeareth to be, 36.

And thus have I founde the 3. exceedes.

Now so to finde the 3 defaultes or wantes, I adde one to eche roote, and multiplye them square; and so of, 102. I finde the square to bee, 10404, and if I subtracte the firste number, whiche is, 10296, out of it, there will remaine, 108, for the firste wante.

Then for the seconde roote, 97, I take, 98, whose square will bee, 9604, out of whiche I abate the seconde number, whiche is, 9493, and there is left 111 as the wante of the seconde number.

Thirly, I take 93 for the newe roote, next above 92. and I finde his square to bee, 8649, from whiche when the thirde number, 8500, is abated, the defaulte appeareth to bee, 149. And thus have I the 3. defaultes or wantes, and also the 3. exceedes, Whiche for ease of comparing, I set in order thus.

A.  B.  C.  A.B. and C. beto:
Exceedes.  95  84  36  ken the order of 
Wantes.  108  111  149  the 3. firste nöbers.

And here I compare the exceedes with the wantes, to see if any 2. exceedes will make up the others want. And I see by a lighte prove, it will not serve.

As for the wantes, I doe not compare them to the exceedes,
The extraction

staffes, for I se that ever one want, is greater then any one staffe. And therfore 2 wantes are farre to greate above any one staffe. And so am I at a state, 

rather. Therefor although that rule bee gener-
ralle, yet where it failleth, this shall you doe. 

Take the 2 wantes, of any 2 numbers, and adde them first together, and then abate them from the thirde number: and if the remainder be a square num-
ber, then haue you gotten your purpose.

Scholar. That will I prooue here. And first I take the wantes, of the 2 firste numbers, which make 219. And that doe I abate from the thirde number 8500, and there remaineth 8281, which as I see, maie be a square number. And therefor I prooue it, in my tables, and I finde it so to bee. And. 91. to bee the roote of it.

Wherefor I saie to the question, that these shall be the numbers of the 3 battailes, as here I haue set the.

The firste battaille. 10404. and his fronte. 102.
The second battaille. 9604. and his fronte. 98.
The third battaille. 8281. and his fronte. 91.
The somme of all 3 battailes. \( \frac{1}{2} 28289. \)

And because these nöbers are not onely square, but also their whole somme dooth agree, with the somme of the 3 seuerall armiues, you maie be sure that thei are well parted, according to the intente of the question.

But because soche questions, haue more difficultie then commoditie, to them that are not mete, to be traveiled in soche marshall affaires, I wil leave that matter to marshall men, and will come to lower waters in warre.

A question of salyng.

A citie should bee sall at, beyng double dichted. And the inner diche. 32. foote broade. And the walle. 21. foote high. The capitan commaundeth ladders to be made
of Rootes.

made of that juste lengthe, that maie reche from the
biter brow of the inner diche, to the topp of the wall
as in this figure C.
is partly expresse
ed.

where the line
A B standeth for
the breadth of the
diche. And the
line B C for the
height of the
walle. Noiue I
demande, what
shall be the length B
of the line A C, the
which here doeth represent the ladder?

Scholar. This figure doth occasion me to remeber
the 32. theo commem of the path the walle, which I faith thus.

In all righte angulled triangles, the square of
that side, which lieth against the righte angle,
is equalle to the two squares of bothe the o-
ther sides.

Whereby I understand that I must multiply those
twoo sides squarly, that is, eche of them by it selfe.
And then adyng those, tow squares together, I must
extrait the roote of that whole nomber: whiche roote
shall be the true lengthe of the slope line.

Therefore, firste I multiply 32. by it
self, and there riseth of it 1024.
Againse, I multiply 32. by it
self, and it yeldeth 1024. These
bothe sommes, beyng added to-
gether, doe make 1465, whiche
nomber maie bee square, because it begins
P.3. neth
The extraction

neth with. S.

Master. It is no square number, as it appeareth at the first sight. For although the first number be 5, yet in such numbers it is requisite, that the second figure should be 2. Else can it not be square: and here, you see, that the second figure is 6, so that it can not be a square number.

Wherefore you shall seek the highest root, that you can find in it, and take that for your purpose.

Scholar. Here is my woode set for the.

And so it appeareth well that the highest root is \( \sqrt{38.6^2 + 1} \), which he is left then a quarter of a foot, above 38 foot and that must be the length of the ladder.

Master. Yet one question more will I propound agreeable to the first frame.

A question of encamping. A capitaine generall hath yenge three armies, in three several battles, in the first 4900 menne, in the second 2401. And in the third 2500. (so that the greatest army is as many as both the other, except one manne) is enforced to joine all three battles in one. But is in doubt, whether he may have good and convenient ground to encamp the in battle so close. Wherefore considering he all 3 battles together, are but double to the greatest of the 3 alone. The capitaine discovering a more ground for his army, so joined in one square battle, is in doubt, what square of ground will serve his purpose. But sure he is, that it must be double to the grounde, that the greatest armie of the 3 did occupy and that was square every wales 210 Soote. Wherefoze his demande is, how many soote square, shall the side of that grounde bee, that is double to the former square platte, whose side was 210 Soote every wale?

Scholar,
of Rootes.

Scholar. Firste I must multiply 21 2, by it self, and so have I the last plate of grounde of 44100. foote, that must I double, and it will be 88200. And out of this number, shall I take the highest square root. For a square square, I se, it is not: by reason that after the even Cyphers, there followeth 2, which is one of those figures, that can not beginne any square number.

Wherefore, seeking for the highest square, I finde it to bee 296, that is almost, 297. foote every wates square. And so moche muste the square side of that grounde bee, which should serve for that whole armie.

And hereby I doe perceiue, the oversighte of many men: which being required to double a square plahte do double the side of it, thinking the mater easily done.

But if thei marke it well, thei maye perceive, that they doe make, by that meanes, a square fower times so bigge as their firste square was. As by this figure, any man maye see.

For if 2 be the side of the square then is the square 4. But if I double the side, and make it 4, the square thereof will be 16, which is 4 tymes 4, and not onely double.

So that the root of the double plahte, should bee the root of 8, which is somewhat lesse then 3, and therefore moche lesse then 4.

Mater. You maye perceiue the same, with the reason of it, by the 18. proposition of the 8. booke of Euclide, as it is before alleged.

But now soe therewe the larger use of this rule,
I demande this question.

There be 2. townes, as Chichester and Yorke, which lye South and North, and betwene them 220 miles. A thirde town as Exeter, lieth plaine Weste from Chichester, 120 miles. I desire to knowe the true distance of Yorke from Exeter.

Scholar. I must set those 3. townes, in forme of a Triangle, with their distances:

As here is represented. Where

A. North for Exeter,
B. for Chichester,
C. for Yorke.

And then acco2

dyng to the rule,

I multiplicate 120, 220, and 220, and it yeldeth 48 400.

These both numbers I shall toynge in one, and so haue 628 000. whose roote is very nigh 250 miles

And that is the true distance of Yorke and Exeter.

By this example I gather, that this rule dooth helpe to Geo. graphic, so2 to drawe the true platter of any country.

Mister. If I should stande in propounding examples of this rule unto you, dyng but one so2 2 every arte and science, and so2 every different kind of commodious practise: if would make a great booke.

And therefore omitting that, till occasion serve other waies, I will proccede to the extraction of Cubike rootes.

Of
Of Rootes.

Of Cubike rootes.

When any Cubike number is propounded, whose roote you should extract.

After the number is written down orderly, you shall set a picke under the firste figure: and under the 4.

and so under every third figure, skipping till 2 figures unpicked.

And looke how many pickes, your number hath, so many figures shall the roote of your nober contain.

Then to begin the searche, for the firste figure of the roote (in this order) you shall looke what may be the roote of the number, belonging to the left picke toward the left hande. And that roote shall you sette by a quotiente line, as you did in square rootes.

And if the whole number over that picke, be a Cubike nober, you shall cancel it all. But if it bee no Cubike nober, then subtracte out of it, the greateste Cube in it, and cancel the whole nober, and set the reste over it: as you did in square rootes.

But considerynge, that you ought to have in ready remembrance, all those Cubike rootes, which be digitas, with the Cubes that they make: for without them you can not proceede in this woork. I thinke it good to set for the herein a table, all those rootes with their Cubes, that thereby you may be the more assured in thyne of your woork. For else a little mistaking, might be the occasion of a greate erroure.

And now for this firste rule I saie, as I saied of Square rootes, this shall be ever more the firste woork, and shall not be repeated in any one Cubike nober. Whereas all the other rules following, shall be so often repeated, as there are pickes in
The extraction

2. And of them this is the firste: that you shall triple the firste roote. And that triple you set under the nerte number, toward the righte hande, before that picke, whiche you did laste ende.

3. Then multylic that triple, by the same quotiente. And set it done under the firste triple: and that number shall be called your divisor.

4. Thirdly, take out a quotient, that maie declare how often the divisor is in the number over it.

In whiche doing, you must have this regard, that betwene that picke that is ended, and the nerte that standeth toward the right hande, you must subtracte 2. other numbers. That is to saie, the square of the laste quotiente, multiplied by the former triple, 10. tymes: and the cube of the same quotiente.

Scholar. This rule is very obscure in words.
Master. Then will I terme it thus.

2. 4. 3.
4. Take the square of your whole quotiente, 300. tymes: and that shall be your divisor: Then feke a newe quotiente, declaryng how often that divisor, maie be founde in the number, that doeth belong to the nerte picke. But so that the square of that newe quotiente, multiplied by the laste quotiente, 30. tymes: and also the cube of that newe quotiente, joyned all in one somme, maie be taken out of the same number. And if you understande this, there resteth no more difficultie.

Scholar. I trust by creple, to understand it better.
Master. Then take you this creple, 26463592 which I shall set done and picke, as I taught you before: and as you maie here see. Where the 3. pickes declare unto me, that the roote will haue. 3. figures,

And then under the picke that is nerte the lest hande, whose number is 26. I finde the greateste Cubike number to bee, 8. and his roote. 2.
of Rootes.

For 27, which is the nerte Cube, is to great.

Therefore I set 2, in the quotiente, and his Cube, being 8. I doe abate out of 26, and so remaineth 18.

whiche I muste cancell: and then standeth the number, as here you doe see.

This is that firste worke, whiche is not repeated.

Then to procede forward, I doe triple the quotiente 2, and so have 6. whiche I shall sett under 4, being the nerte number, on the righte hande of the pricke that is ended.

And that triple must I multiply by the firste quotiente, whereby amoughteth that number, that must be the divisor: and it is in this worke 12.

whiche must be set under the same triple: as here I have placed it.

Then shall I take for a newe quotiente, declaring how often tymes 12. may be founde in the number over it, that is 18. 4. And I see it may be in appearance 15 tymes, but more then 9. you shall never take for a quotiente: wherefore it appeareth, that I may boldly take 9. whiche I shall sette in the quotiente with the firste 2.

And then shall I multiply. 12. whiche is the divisor, by 9. and thereof commeth 108. to be sette under 184. beneath the line, whiche shall evermore be drawen under the divisor.

Now must I take the square of my laste quotiente 9. (whiche is 81.) and multiply it by the triple of the former quotiente (that is by 6.) and so have 486. to be sette on place more toward the right hande.

\[
\begin{array}{c}
2 \\
18 \\
26 \\
108 \\
16389
\end{array}
\]

\[
\begin{array}{c}
184 \\
256 \\
486 \\
729
\end{array}
\]

Last
The extraction

Lauce of all, I shall multiply the laste quotiente Cubikely: and that makest. 729, whiche must be set, yet one place more toward the right hand, that is to saie, under the nexte picke. And then shall I addde those 3 sommes into one: wherby will rise. 16389, to be subtracted out of. 18463, and so will remaine over that picke. 2074.

And the woorkke of that picke is done.

This order of worke, if you marke well, you have learned the whole arte of extraction of Cubike rootes.

For how greate so ever your number be: you shall not have any newe kinde of woorkke.

But yet because I did teache you before, the same woorkke in other woorkdes, I will woorkke the same example again, accorpyng to these woorkdes.

And firste, after that the number is set downe, and the first Cubike rootte taken, and the Cube abated. Then take the square of that roote. 300. tymes, that is in this example. 4. tymes. 300, whiche makest. 1200, and that shall be your divisor. This number, and all other in this woorkke, shall you set downe so, that the firste number, shall be under the nexte picke, toward the righte hande.

Then seke your quotiente, with the former cautele, and it will be. 9. Wherefore multiplyng. 1200 by 9, there will amonite 10800, to be set under the line.

After this, I shall take the square of. 9 (whiche is the new quotiente) and multiply it by. 2. Whiche was the laste quotiente before) 300. tymes. So multi I multiply 81 by 60, and it will make. 4860. whicke I place orderly.

Then set I doune the Cube of the quotiente, whiche maketh
of Rootes.

maketh. 729. And so are the 3 numbers placed, and agree with the former woork, in all things, saue in 2 points. For here the triple of the quotiente, is not set done, but kepte in memorie. And again, here are diverse cyphers, which are not in the former woork.

Scholar. Sir, I perceive, that the Cyphers doo nothing else, but set the numbers in their due places. And the triple of the quotiente, is supplied in woork by 2 multiplications. First by 300, and then by 30. So that it is all one in effecte.

And by the one woork, I understand the other the better: when I compare them both together. But yet I praie you, ende the woork that you began.

Master. To continue that woork, first I must set done the numbers, as they should remaine, after 16389, is abated out of 18463, and then will they stande thus.

Then shall I repeate the former woork, by setting done the triple of all the quotiente, which will be 87, and that must be placed under 45.

Herte that I shall multiply that 87 by 29, and there will come 2523, whiche must be the multipli.

Therefore I seek for a new quotiente, that may theylke me how often 2523 is contained in 20745. And it will bee 8.

That 8 doe I set in the quotient and by it I multiply 2523, and it giueth 20184 which I sette done, as here you see.

Then doe I multiply that quotient squarelvy, and that will be 64. Whiche I shall multiply by the triple, that is 87, and there will amounte 5568, to be set one place more toward the right hande.

D.s. Last
The extraction

Last of all, I must take the Cube of 8, that is, 512, and it shall bee sette yet one place more towards the righte hande.

And then by addition, I shall byng all into one number: and it will bee, 2074592, which is equall with the whole number aboue, that is uncanned. And therefore if I abate the one out of the other, there will remain nothynge.

Wherefore I see, that the firste number, is a cube Cubike number. And his roote is, 298.

Scholar. I have marked you so well, that I trust to doe the like, without errour.

But I prate you woorke this laste parte also, by your seconde rule, as you did woorke the other: that I maie see the due agreemente of thembothe: and also perceive the righte use of this woorke, the better by that other forme.

Waster. I must in that case sette doune the numbers, as they were set in the other woorke. And then I shall multiplie the quotient, which is, 29. by it self squarely, and it will make 841. which must be multiplied by, 300. And so there amounteth, 252300, to be sette doune, as here you see.

Then I shall seek out a quotient, declarynge how often 252300, maie bee found in 2074592. And that quotient will bee, 8: whiche I set in the quotient roome, with the other numbers.

And then I doe multiplie the divisors, by the quotient, and thereof riseth 2018400 whiche I set under a line, as you maie see.

Nexte that, I doe multiplie the newe quotient, by it self,
of Roots.

self squarely, whereas commeth, 64, and that square of the last quotient, I shall multiple by, 870, which is, 10 times the triple of the former quotient, 29; and thereof commeth, 55680, which I set done also orderly.

Laste of all, I multiplye, 8, (that is the laste quotiente) Cubike, and it maketh, 512, which also I set done in common order.

And then shall I adde them all together. And so haue I the same sonne, that I had before in the other former woork, and it is, 2074592.

Scholar. I neade no more instruction for this: I thinke my self so cunning, by occasion of your examples, whiche you haue woulde so in double sonme.

Master. That woulde you proue, by this number 47832147.

Scholar. Firste I shall pricke it, as you taughte me, omitting still, 2, numbers.

And then out of the number over the laste pricke, I shal seke out the Cubike roote, and abate the Cube thereof, out of the same number, and set the remaine once over it, cancelling the reste.

And so in this number, I finde in, 47, the greateste Cube to be, 27, and the roote of it, 3. Wherfore I abate, 27, out of, 47, and finde the reste to be, 20, therefore I cancell, 47, and set, 20, over it. And the, 3, whiche is the roote, I set in the quotient. And so is the first woork cande.

Then doe I triple that quotiente, and it maketh, 9, whiche I set done under, 8.

Again I multiple that, 9, by, 3, and it yeildeth, 27, whiche I set under the triple, and take it for my divisor.

Wherfore I shall now seke a quotiente, that maie
The extraction

declare how often. 27. is in 208
and I see, it will bee. 7. tymes.
Therefore I sette doune. 7. in the
quotiente: and by it I multiply 27
and it maketh. 189. whiche I set
under the line: and then I dooe
multiply. 7. by it self, whiche
maketh. 49. 4 that square doe I
multiply by the triple of the for-
mer quotiente, that is, by. 9. and it yeldeth. 441. whi-
che I set one place more toward the righte hande.

Last of all, I take the Cube of. 7. whiche is. 343. and
that doe I sette doune, yet one place more toward the
righte hande.

These. 3. sommes beyng added together, doe make
23653.

Master. That will be hardly abated out of a let-
sier somme.

Scholar. I see now my errour. I must take a lesse
quotient: whiche thyng I might have perceived by the
seconde number. For thet twoo wcer to great, before
the thirde was added.

So that I should have taken but. 6. for the quotiente
And then would the firste number have been but 162
and the seconde. 324. and the
thirde. 216. but that their pla-
cyng would make them to be of
other values, save the last of the.

Therefore, I set every one in
his due roome: and adde theim
together, and there amounteth
19656. to bee subtracted out of
20832. and the remainder will
be 1176. And thus is that pricke
with his wooze canded.

Then for the nexte pricke, I repeate the same very
sozme
of Rootes.

some of worke againe. First settinge downe the triple of the whole quotiente, which is. 108. so that it shall stande under. 11761. or under. 761. accountynge figure for figure.

That triple must I multiplicye agayne by the whole quotiente. 3. and it will make. 3888. whiche number I muste take for my divisor.

Wherfore I seke how many times, I may finde that divisor in. 11761. and I see, it will bee. 3. tymes. Wherfore I set. 3. as my quotiente, in his due place; and by that quotient I do multiply. 3888. and so haue I for my firste number. 11664.

Againe I doe multiply the laste quotiente. 3. squarly, and so haue I. 9. whiche I shall multiply by the triple of the former quotiente, and it yeoldeth. 972. that shall be set more higher the right hande, by one place.

Thirdly, I take the Cube of. 3. whiche is. 27. and that doe I set yet one place more towarde the righte hande.

Then doe I adde those 3 sommes into one, and they make. 1176147. whiche is equalle somme, with all the numbers over it, that be uncancelled.

Wherfore I saye that. 47832147. is a Cubike number, and the Cubike roote of it is. 363.

Master. How doeth the order of teachynge require, that I should instructe you, how to extracate the roote in and the higheste Cube roote, out of any number, that is not a ber not Cub true Cube. As this number for example maie serve. bike.

694582591.

Where firste I muste extracate the higheste roote, as I taughte you, for the higheste Square rootes, in numbers that are not square; and then shall I note the res...
The extraction

mainer: which I shall set for the numerator. And his denominator shall be found, as I will tell you anon. But first do you work the example, as his night's roote in whole numbers.

Scholar. I set it done, and pricke it, and finde the greateste cube over the laste pricke to bee \[82\]
\[694\] 58 2951 (8. 512). and the roote of it is 8.

Wherefore I set done 8 in the quotiente. And I abate 512, out of 694. and so resteth 182, and the former 694 cancelled.

Then to proceed, I must triple that roote, 8, and it maketh 24. whiche 24. I set under 1825. And then I doe multiply that again, by the quotiente 02 roote, 8 and it maketh 192. to be set under the said triple, 24: as the divid. For which I take a newe quotient, and it will be 8. That 8. I set in the quotiente place, and by it I multiply the divid 192. and there riseth 1536. to be set under the line, in convenient order.

Next I multiply the quotiente squarely; whiche yeldeth 46. and that square I multiply again by the triple, and so hau' I. 1536. also. But this must stand more forwardly by one place.

Last of all I take the cube of the quotient 8. and that is 512. whiche I set under the other twoo sommes, and that by one place more forwardly.

Now gathering all these, 3 comes into one, thei will make 169472 which I shall abate out of 182582. and so remaineth there 13110. And that pricke with his worke cande'd.

Therefore having one other space to work, I must repeat the same order of worke again, by tripling the whole quotiente.
of Rootes.

quotiente. 88. and that will bee. 264. And againe I must multiplie that triple number, by the said quotiente, and it will make. 23232. which shall bee the divisor.

Wherfor I take a newe quotiente, which is easily per ceived to be. 5. That. 5. doe I set in the quotiente, and by it I doe multiplie the divisor 23232. and there amounteth 116160. as the firste number, to bee set under the line.

Againe I shall multiplie the quotiente squarely, which giueth. 25. and that square shall I multiplie by the triple. 264. and so will there rise. 6600. to bee sette, as the seconde number under the line: and one place more for wardly, toward the righte hande.

Last of all, I shall sette under them bothe, and one place more toward the righte hande, the Cube of. 5. which is. 125.

And then shall I addde all those. 3. somes together of whiche commeth. 11682125. to bee abated out of 13110951. and so the remainder will bee. 1428826. Wherby I see, that the firste number that was pro pounded, I mean. 694582951 is no Cubike number, but the greateste Cube in it is. 693154125. and his roote is. 885.

And so, I see, all other numbers of like kinde must bee wrougthe.

But now for the remainder, how shall I dooe to hyringe it into a fraction, that maie aptly express the r. Aforesaid roote in that sorte?

Master. There bee as many waies, as there bee writers almooste, for every manne deviseth, how to
The extraction bynge it moste nigheste to a true roote, is any loche were: whereof Cardane his rule is this.

Multiplie the roote squarely, and againe by 3, and that nomber shal be the divisor unto the remainder.

Where he might have used more plainesse in wordes, if he had saide: and that nomber shal be the denominator, to the remainder. Wherefore as here your roote is 885 so is the square of it 783225 and the triple of that is 2349675. So would that fraction bee

But how nigh this doeth go to the truth, I leave it till another tyme.

Scheubell. Scheubelius doeth allege an other reason, and inforreth an other order, diverse fro this, and loche as impugneth this, sayng:

Triple the roote, and the square of it also, and adde bothe those nombers together, and one more: and so have you a denominator for your numeratour.

The denominator euermore is understood to be the remainder. By whiche meanes the fractio in this worke would bee: which is a lesser fraction by a good deale, then is the former fractio, after Cardanes forme.

But because at this presente, I maye not spende so moche time, to scan their severalle opinions, wherein eche of theim, pleaseth himselfe well: the one alleging demonstration (whiche scarcely serueth) and the other namynge it a secrete, as it is worthie to bee: I will procee to a thirde wate, moze certain than ether of these bothe. And that is by addition of certain Cyphers, to the remainder, in loche sorte, that thei musste all waies bee ternaries, as 3.6.9.12.etc. And then sarche
of Rootes.

Searche forward with the like order of worke, as you used before.

In this manner of practice, looke how many prickes your ciphers hath (or els how many ternaries of Ciphers, there be set to your number) so many figures shall the numerator of your fraction contain, and the denominator shall evermore contain 1 more. Whereof the latter onely shall bee an unitie, and all the other shall bee Ciphers.

That is to saye, that if I adde but 3 Ciphers to the number, the fraction shall contain certain 100 partes. And if I adde 6 Ciphers, it shall expresse 10000 partes. So 9 Ciphers maketh the denominator to bee 10000 partes: And 12 Ciphers giveth 100000 partes.

For example, I will adde to our latter number that remained 12 Ciphers. And then will the number be 1428826,000000000000, unto which I set no more prickes, then scrutiny the ciphers, because I have paied all the other prickes, in my former worke.

And now to continue my worke, I shall triple all the former quotiente, and it will be 2655, whiche number I shall place, as here you see it set. And then shall I multiply that triple, by the former quotiente, 885, whiche will yield 2349675, to be set under the saide triple: as I have sette it here also. And this number shall be the divisor.

Then shall I seeke for a quotiente, whiche can bee none other then 6: wherefore I sette 6, in a quotiente line, and by that 6, I doe multiply the saide divisor 2349675, and it giveth 14098050, to be the firste number under the line.

After that, I take the square of the saide quotiente, whiche is 36, and by it I multiply the triple, 2655:

P.s. wherby
The extraction

Wherby is made 18064984
95580 to be the first number
1428826000000
2656
2349675
2349675
14098050
95580
216
1410761016

Last of all, for the third number I take the cube of the said quotiente which is 216, and place it as you see, with this figure under the picke.

Then doe I add those 3 numbers into one, which maketh 1410761016. And that by the subtraction out of 1428826000, doth leave 18064984. And so is the woorkke of the first picke ended.

Whereby it appeareth, that the fraction is somewhat more than \( \frac{1}{50} = \frac{1}{2} \); as it shall bee tried better, by the woorkkes that shall ensue.

Therefore I proceede to the next picke. And firste I triple that whole quotiente, which yieldeth 26568, to bee set, as it is often before repeated, and therefore nedeth not hereafter to bee tediously rehearsed.

That triple I shall I multiply again, by the whole quotiente (as here I have sette it in woorkke, because the number is greate, and not easily wroughte by memozie) and it doe I set in his due place, as you see.

But then seyng that divisor is greater then all the number over it, I shall set a cypher in the quotiente; in token that the divisor, can not be abated ones out of the number over it.
of Rootes.

It. And so is the 402 ke of that prick e canded, without any more trouell.

Wetherfore to go for ward, I triple all that quotiente and let it bounc, as the rule would, as here is seen.

| 1594819256457 |
| 265680 |
| 23528620800 |
| 164700345600 |
| 13018320 |
| 343 |
| 16470164743543 |

Then dooe I multiplye that triple, by the whole quotiente, whereof cometh. 23528620800, and that shall bee the divisors. And the quotiente for it will be 7.

So then I multiplye that divisors by 7, there will amounte 164700345600, for the first number to be set under the line.

And so the nerte woo2ke, I shall multiplye 49, (which is the square of the newe quotiente) with the triple of the former quotiente, and it will bringe forth 13018320, which shall bee the seconde number, to be set under the line.

The thirde number shall bee the Cube of 7, which is 343.

And those 3, sommes added together, will make 16470164743543, which is to bee abated out of 18064984000000, and then shall there remain 1594819256457. And so haue I canded 3 prickes of the Cyphers. And thereby maye sate, that the fraction is \( \frac{667}{555} \) and somewhat more : That is somewhat more then \( \frac{1}{2} \).

Scholar. I see by the fraction, that it is \( \frac{1}{2} \) and \( \frac{1}{3} \), beside
The extraction
beside the quantitie of the remainder. But I praze you sende the woole of that other pike, which dooth remaine.

Matter. I muste triple all the quotiente, whereby will rise. 2656821, whiche muste be multiplied by the said quotiente: and thereof will proceede the dividus, being 2352899275347. And his quotiente will bee 6.

Wherefore firste I set. 6. in the quotiente line, with the other numbers: and then doe I multiply the dividus by that quotiente, and it beyngeth for the firste number to be sette under the line.

| 183078734793024 |
| 159409260 |
| 13284105 |
| 21254568 |
| 21254568 |
| 2352899275347 |

| 14117395652082 |
| 95645556 |
| 216 |
| 1411740521663976 |

And again the square of 6, beynge multiplied by the triple, will yelde. 95645556: which shall bee the seconde number under the line.

The thirde number shall be. 216. because it is the Cube of 6. And those 3. numbers beynge added together, doe make. 1411740521663976. to be abated out of 1594819256457000. And dooth there remaine. 183078734793024.

Wherefore
of Rootes.

Wherefore it doeth appeare, that beside the first. 3 numbers of the roote, that is 885. the reste (that is 6076.) standeth for the numerator of a fraction, and the denominator unto it is. 10000.

So that the higheste roote is. 885, beside the fraction that doth remaine: which would make but \(\frac{1}{3}\) of a.

Scholar. This is a sufficiente precisenes. And so I judge it sufficiently taughte.

Wherefore I praise you propounde some questions, that doe require this arte, for their solution.

Master. I am contente. And let this be the first.

The Grecians given to idle bankeeting, and soche a question like vaneconnell, did procure thereby soche mortalle sicknesses: that the quicke were scarce hable to burie the deade. Wherefore consultaunge with their Goddes, for redresse thereof, they received answere, that when they would double the Altare, which was of Cubike somme, they should bee delivered from that plague. Meanynge that learning is a due meane, to deliver realmes from plagues and enowmitie. But to the question, what saie you? If the side of a Cube be. z. foote (as that altarke might bee) how many foote shall the side be of that Cube, which must be double unto it.

Scholar. This I consider. That firste I must finde the quantitie of the Cube, that is propound. And then shall I double that quantitie. Thirdly, I must extracte the Cubike roote, of that double number.

So in this question, the side of the knowne Cube is 3. and therefore the whole Cube is 27. whose double is 54. And the Cubike roote is 3. and \(\frac{3}{2}\) by Cardanes rule: That is 4. whiche is plainly falsel, for 4. is the roote of 64. and not of 54. But by Scheubelns rule, it will be. \(\frac{3}{2}\) that is. \(\frac{3}{2}\) almoste: which is noche nigher the truthe. For \(\frac{3}{2}\) multiplied Cubikely, doeth make. 52. \(\frac{1}{2}\), which is to little by a good deale, that is by. 1 \(\frac{1}{2}\).
The extraction

Whereas \( \frac{32}{17} \) doeth make a letter somme: that is to say but \( \frac{51}{21} \) and so wanteth. \( \frac{24}{9} \) and although bothe these sommes goe nigher to the truthc, then Cardan's rule, whiche musteth. I o. wholly; yet make it be easily seen, that Scheubeline's rule is not so good, as he would it were. And the worte here, for the addyng of that one more.

Master. You are lepte verie sodenly from a schollar, to a comptroller. And yet I can not but praise your diligent observing of soche thynges.

Prove now by the Cyphers, how it will frame.

Scholar. I sette doute the nomber with. 6. Cyphers, and picke them thus.

Then doe I take the greateste Cubike nomber in. 5 4. whiche is. 27 \( \frac{540000000}{3} \) and that I doe abate from 5 4. and so resteth. 27. the roote of the Cube is. 5, whiche I sette in the quotiente line.

And then I triple. 3, whiche makest. 9, that must be multiplied by the quotiente againe, and so commeth 27. to be the divisor. And his quotiente remeth to be. 9.

Wherefore woozyng with it, the firste nomber is. 243, and the seconde is. 729, that is. 81, multiplied by 9, whiche is the triple.

Againe, the Cube of. 9, is. 729. And all thet together, dooe make 32319 whiche seime is to greate, and therefore I must take a letter quotiente. As I might have per-

\[
\begin{array}{c}
27 \\
540000000(38/9 \\
27 \\
216 \\
576
\end{array}
\]

I receiued well enouch by the second nober, if I had marked it in time.

But now amendyng my over lighte, I take. 8, for the quotiente.

And woozyng with it I see, the firste nomber under the line, will be
of Rootes.

bec. 216, and the seconde. 576. And here all ready I espie my oversights again.

Therefore I take 7 to be the quotiente. And by it I multiplye the diuisor, and so haue I, 189, for the firste number.

And for the seconde number, I doe worke with 49, which is the square of the quotiente, multiplied by 9, that is the triple; and it yeeldeth 441.

Thirdly, I take the cube of 7, which is 343. And then addynge all 3 numbers togethers, I finde the somme to bee 23653, which is to bee abated out of 27000, and so resteth 3347. Wherby I see, that 3347

With somewhat more is the roote that I should finde.

But for farther trial, I triple all the quotiente, and finde thereby 111, which I multiplye by the same quotiente again, and so commeth 4107, to bee the diuisor. And his quotiente will bee 8, as it semeth; and so the first number will bee 32856. And the seconde shall bee 7104, but those 2 are to great, as it is manifeate all readie.

Wherefore I take 7 for the quotiente. And by it multiplying the diuisor, there riseth 28749.

And for the seconde somme, there is founde 5439.

And for the thirde some 343.

All which 3 sommes togethers in one, dooe make 2929633, and that beeing abated out of the higher somme 3347000, dooth leaue 417367.
The extraction

Wherefore I make boldly saie, that the fractio[n is \(\frac{27}{77}\) and more, by the po[son of the remainer, which is nigh \(\frac{1}{33}\). And it is sone seen that \(\frac{27}{77}\) are equale to \(\frac{1}{3}\); where

fore \(\frac{27}{77}\) shall be more then \(\frac{1}{3}\). And so dooeth Scheu[belin's rule erre more, then I thought before.

So is your question answer[ed, that the side of the double Cube, shall be \(\frac{3}{2}\) foote and \(\frac{27}{77}\) and \(\frac{1}{3}\) of \(\frac{1}{27}\).

Of the roote of fractions. Master. For the rootes of fractions, I shall nede
to saie no more but this: that if the numerato[r and de

nominato[r bothe be Squares, or Cubes, &c. then may you

finde in that fractio[n the like roote. But if any of bothe
do[e swarue from that name, then hath that fractio[n

no roche roote.

As \(\frac{1}{3}\) is another Cube nor Square, because his partes

do[e not agree in Square name, no; in Cubike name: al-

though the numerato[r be a Square, and the de

nominato[r a Cube.

Scholar. That dooeth appeare reasonable, at the

the firste sighte.

Master. Then seeing you are so readie in learn-

nyng; answer me to this question.

A Cone of fife inches diameter in the mouthe,
doeth shotte a bollet of twentie pound weighte; what

weighte shall that bollette have, that steath for a
gonne of \(\frac{1}{2}\) inches in the mouthe?

But to helpe you in this question, and in all roche

like, you shall marke well Euclide his sayng, in the 18

proposition of his 12 booke, w(ich he is this.

All Globes bire together triple that propor-
tion, that their diameters doe

So in this example, the proportion of the diameters

beyng as \(\frac{1}{4}\) to \(\frac{1}{6}\), \(\frac{1}{2}\) as \(\frac{1}{7}\) to \(\frac{1}{3}\), I shall tripe[ it, and

then have I the proportion of their Globes.

Wherefore
of Rootes.

Wherefore I sette the 3. fractions thus. \( \frac{7}{3} \cdot \frac{7}{4} \cdot \frac{7}{8} \) and thynk to make \( \frac{71}{8} \). that is, \( 12 \frac{3}{8} \). And so is the proportion of the Globes, as well in weighte, as in biginesse.

Therefore I must multiplye 20. that is the weight of the lesser bollette, by the numeratoz of the proportion, and divide it by the denominatorz.

And so shal I haue \( 25 \cdot 4 \frac{1}{7} \) for the weighte of the greater bollette.

Now prove ye the like proporcions. Remembering that Cubes also, as well as Globes, doe brack triple proportion, in comparison of their sides. As you learned before by the 19. proposition, of the 8. booke of Euclide.

A Cube of 8 zoll of 4 inches square, dooth weighte a question 7. pounde weighte. What shall a Cube of 8 zoll of 9. of 2. Cubes. inches square, waie?

Scholar. The proportion of the sides is as \( \frac{9}{4} \) whiche I must set downe thrise, and multiplye them together, as fractions should bee. And so will it bee thus, \( 2 \cdot \frac{9}{4} \cdot \frac{9}{4} \), that maketh \( \frac{729}{64} \).

Wherefore I multiplye the weighte of the lesser Cube, byng \( 7 \) by \( 7 \cdot 2 \cdot 9 \), and it maketh \( 510 \cdot 3 \), and that doe I divide by \( 6 \cdot 4 \), and so finde \( 79 \cdot 2 \), whereby I maye knowe, that the weighte of the greater Cube, is 79. pound weighte, and very nighte \( \frac{1}{2} \).

Master. These 2. questions dooe teache you, rather the proportion of Cubes, then the use of the rule. Wherefore to make the queestions more agreeable to this rule, I propounde them thus, in backer order.

A bollette of 70 of 7. inches diameter, dooth waie 27. pounde weighte: what shal the diameter to that bollette that shal waie 125. pounde weighte?

Scholar. I praie you answer to it your selfe, that I maye see the apte forme of applyinge suche questions.
The extraction

to this rule.

Master. As the Cubes are in triple proportion to the sides, so are the proportions of the sides, to be found by triple division: that is to say, by seeking the Cubike rootes of the 2 terms of the proportion.

Wherefore I doe firste set downe the terms of the proportion of the bollettes, thus: \( \frac{\sqrt[3]{25}}{\sqrt[3]{18}} \). And I see, that the Cubike rootes of 125, 18, 5, and the like rootes of 27, 18, 3, where numbers I shall set in the room of the 2 others, thus: \( \frac{1}{7} \). And thei declare the proportion, be-
twene the diameters of the 2 bollettes. Whereof one that is the lesser, is knowne to be 7. Therfore I multi-
plie that, 7 by 5, whereof commeth 35, and that, 35, doe I diuide by 3, whereof glueth. \( \frac{11}{3} \).

Wherefore I saie, that if 7, inches bee the diameter to a bollette of 27. pounde weighte, then, 11, inches and \( \frac{1}{3} \) Shall be the diameter to the bollette of 125. poudre weighte.

Scholar. The proofe of this had neede bee certain, seyng the woorkke is obscure, to the common judge-
mente.

The proofe. Master. You saie well. And this is the very or-
der of proofe for it. Multiplie bothe these rootes Cubi-
kelie. And if theire Cubes be in soche proportion as their weightes bee (that is to saie in this exaële as \( \frac{11}{3} \)) then is the woorkke good: els not.

Scholar. That must needes bee so. And therefore will I proue it so in these numbers.

And soz that cande, firste I multiplie 7. Cubikelie, and it glueth 343. Then I multiplie 11 \( \frac{1}{3} \) Cubikelie, and it maketh 11463. But now seyng the one number
is a fractio[n], I wil soz case tourne the other into a fractio[n] of the same denomination: and it will bee \( \frac{9281}{303} \) in whiche.2. fractio[n]s, the proportion muste consist be-
twene the numeratours. So that thei bothe seyng divided by one common number, "tucke come to this
fraction
of Rootes.

\[
\text{\textcopyright} \quad \end{align*}
\]

And so I see it will be: for the letter being divided by \( \frac{3}{4} \) will yeeld 27. And the greater divided by the same \( \frac{3}{4} \) will giue \( 1.25 \). So that by trial, that woork is approved good.

Master. I will now proue your cunninge, in a newe question, whiche Brasiers often tynes, have occasion to use: as thus.

I have a dice of Brass of 64. ounces of Troy weighte, whose side is 3. inches and \( \frac{1}{3} \) and would have an other dice of the same mettall of 18. pounde weighte.

My demande is: what shal be the side of the dice?

Scholar. This question must firste bee reduced to one kinde of denomination in the weightes, and then will it be moare apte to be auestioned.

Wherefore I shall tourne 18. pounde into ounces, multiplyng it by 12. and it will be 216.

And then I consider the proportion, that is betwene those, 2. numbers of weighte 64. and 216. and it is certainly \( \frac{3}{4} \), or \( 2 \frac{1}{4} \) out of whiche proportion, I must extracte the Cubike roote, as I male easily doore, lying in the the numerator, and the denominator, are Cubike numbers.

And so is their roote \( \frac{3}{4} \): whiche is the proportion of the sides of the two dice.

And lying the side of the letter dice, is knowne to be 3. inches and \( \frac{1}{3} \), the other his side must be in Sesquialter proportion to it, that is, \( \frac{3}{4} \) : whiche is wrouht also thus. I multiplye, \( \frac{3}{4} \) by 3 and it maketh, 1 \( \frac{1}{3} \) whicher I shall divide by 2. and there commeth \( \frac{1}{3} \).

Master. Yet one question more: I will propounde to gie you occasion, to understande the apte consequene of masse of diverse stuffe.

And for that purpose, I suppose this proportion in weighte, to bee betwene masse of one biggenesse.

D. y. That
The extraction

Examples of rates for weights.

<table>
<thead>
<tr>
<th>Stone</th>
<th>Woodc.</th>
<th>Lace.</th>
<th>Lead.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>150</td>
<td>280</td>
</tr>
<tr>
<td>1/2</td>
<td>100</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td>280</td>
<td></td>
</tr>
</tbody>
</table>

That if I compare wood and stone of one quantitie together, the stone shall weigh more than the wood by 1/2.

Like waies pson to be heuer then stone by 1/2.

And brasse to bee heuer then pson by 1/2.

Lade to be heuer then brasse by 1/3.

All whiche rates, although the be taken for examples, and not of truthe, yet thereby may you learne, how to woorke with true rates, set in a like table.

And now for the use of this table, take this question.

I would haue 5 weightes of Cubike for me, made of these 5 stuffes.

The weighte of the woode shall be 28. pounde.

The stone 6. pounde.

The pson 112. pounde.

The brasse 224. pounde.

And the lade 448. pounde.

Of all these I haue but the pson weighte: whose side, 02 Cubike roote is 12. inches 1/2.

And my desire is to knowe, of what quantitie the sides of all the other weightes shall bee.

Scholar. The question is pleasaunt: and yet some what harder then the other.

Master. The table will helpe you fully, so that you cosefere it well, with that you haue learned before.

But because I have litle leiser, to spende moche tyme with you ( saue that sale to your furtheraunce dooth make me partly to forsette my owne business), therefore will I leue this question to your self, to be aunswered at your laisure.

And so in all the rest, I must posse it over: and giue an ize to suche maters, that touche me more nighte: and
and weighe more heavily, then all sorne weightes, by 2 o. solde.
Wherefore, touching all the rootes of compound numbers, you shall at my hand now, have no private declaration. But soche as you have learned all rede.

Of compounde rootes.

If the number bee compound, or of square numbers, or of cubike numbers, then accordingly as the composition is, so shall you draw the root: and without one of these two there can bee no composition.

Wherefore to begin with the smallest compound number in that zoote, which is a square of squares, you squares of shall firste extrace the square roote, as you have learned before. And out of that roote (whiche must needs bee a square number) you shall extrace his square roote also: and that roote is the zenzizenzike roote, of the firste square of squares 02 zenzizenzike number.

For example take 14641, whose square roote is 121 and that same roote is it self, a square number: and hath for his roote 11.

Wherefore I male saie, that 11 is the squared square roote, or the zenzizenzike roote of 14641.

Again 8503056 is a square of squares, and therefore a square number. And his square roote is 4 2916, which is a square number also, and hath 54 for
The extraction

his roote.
So that, 5 4, maie well bee called the zenzizenzike roote of 8 5 0 3 0 5 6.
And so shal you woorke, with all of that name.

But and if the number be compounde, of 3 zenzizenzikes, 02, 02, Squares, as a Square of Squared squares, 02 a zenzizenzizike (which some men soo shothnesse, call zenzizenzizenzike). Then shall you draue firste the Square roote, and then the Square roote of that roote, and thirdly the Square roote of that laste roote.

As for example. 6 5 6 1, is a Square of Squared squares. And his firste roote is 8 1, which is also a Square number, and hath 9, for his roote. That 9, likewaies is a Square number, and hath 3, for his roote.

So that the zenzizenzizenzike roote of 6 5 6 1, is 3.

And for these sortes of numbers, I shall not nede to state for any more explication, or examples; seeing the matter is plain.

Now for compound Cubike numbers, you shall under stande the like sortes.

If the number bee a Cube of Cubes, you shall firste extrac te the Cubike roote. And because that roote is a Cubike number also, therefore shall you take the Cubike roote of it. And that seconde roote shal bee the Cubicubike roote of the firste number.

As for example. 5 1 2, is a Cubike number, 02 a Cube of Cubes. And his Cubike roote is 8, which is, againe is a Cubike number and hath 2, for his roote.

So that, 2, is the Cubicubike roote of 5 1 2.

Likewaies, 1 0 0 7 7 6 9 6, is a Cubicubike rober, and his firste Cubike roote is 2 1 6, as you maye easly perceive by these woorkes: where I haue sette the order of extrac tion of his Cubike roote, which is 2 1 6. And that 2 1 6, is a Cubike number, you maye not to doubt,
of Rootes.

\[ 816 \quad 2816 \]
\[ 77696 \quad 77696 \]
\[ 63 \quad 6 \]
\[ 1323 \]
\[ \]
\[ 7938 \quad 1264 \]
\[ 2268 \]
\[ 216 \quad 1264 \]
\[ 81696 \quad 1264 \]

By this you may judge of Cubicubes Cubicubikes Cubikely, or Cubes of Cubicubes, that in them cubikely, you shall first seek their Cubike roote: And then the Cubike roote of that roote. And thirdly the Cubike roote of that roote againe. And so have you the Cubicubicubike roote of that like number.

The thirde wise of composition is, when Squares The thirde and Cubes be compounded together: as Zenzi Cubes, Zenzi composition, Zenzicubes, Zenzi Cubicubes, or soe like, as it happeneth diversely.

In all these you shall as often abate the Zenzi roote, as that name is in the composition, and so have waies of the Cubike roote.

So that in a Zenzi Cubike, you shall eronate like the Zenzicube, Square roote: and out of that Square roote, you shall eronate the Cubike roote.

As. 64 is a Zenzi Cubike number, whose Square roote is 8, and that. 8. is a Cubike number, and hath. 2. for his roote.

So. 531441 is a Zenzicube: Whose first Square roote is. 729. Which number is a Zenzicube, I hath for his Square roote. 27. And that number is a Cube, and hath for his roote. 3. Wherefore I make lustly saie, that. 3. is the Zenzicubicube roote of. 531441.

But as I laid bfore, that I might not stale long at
The extraaktion

at this presente, so the use of these greate numbers is rare in practise; and therefore I will overpaule them, for this tyme.

And yet for your aile in this mane season, I have here drawen a table, whiche mate bee called the table of ease: in whiche you have greate plentie of these numbers, with their rootes in diverse kindes.

The table it self is so manifeste, that it neadeth no declaration: if you have not forgotten, what you learned before.

And if you liste to enlarge this table, you mate casily doe it, multiplying the numbers still by their rootes, whiche bee set over theim, in the hedde of the table. And so mate you make it to extende infinitely: whiche shall ease you wonderfully, in the extration of any kinde of rootes. For which at some other time if my leisure serve me better, with quietnesse, I will give you more specialle rules.

And also I counsell you, well to examine this table, and trust not to my calling. For haste and other troubles, mate often times cause errore in supputation.
of Cosike nombers.
Of nombers denominate.

But have I lightly over run the mote Nombers common kindes of nombers Abstraitte, contratte.
And now refeth the treatice of nombers Contratte, or Denominate. Of whiche kinde there bee some called nombers denominate vulgarly: and other bee called nombers denominate Cosikeely. And a thirde sort ther is of nombers radicalle, whiche commonly bee called nombers irrationalle: because many of them are soche, as can not bee expressed, by common nombers Abstraitte, nother by any certain rationalle nomber. Other men call them more aptly Surdenombers.

And although many menne would not accompte them, with nombers denominate, yet I maie inly doe it, for that they require a reduction to one denomination, if they have severalle signes of quantites, as you shall heare hereafter. And those nombers never goe alone, without some other signe, and name of rooted quantitie, annexed to them.

Of the first kinde of nombers denominate, whiche are vulgarly denominate, as. 1 o. shillinges, 1 o. men 2 o. sippes, 1 o. shepe, 1 o o. yeres, and soche like, I will speake nothing in this treatice. But of the other twoo kindes I will somewhat write, for youre learning and contention.

Scholar. Sir, I am moche bounde unto you: And therefore remit all to your owne discretion and good will. Trustynge so to applicke my studie, and empiole my knowledge, that it shall never repent you of your curtese in this behalfe.

Waster. Then marke well my wordes, and you shall perceve, that I will use as moche plainesse, as I maie, in teachynge: And therefore will beginne with Cosike nombers first.
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Of Cosike numbers.

Numbers Cosike, are soche
as bee contrarie into a de-
nomination of some Cosike signe
as i. number. i. roote. i. square
i. Cube. &c.

But as in the use of theim, there bee
certain figures set so to signi-
ifie them: so I think it good to
expresse unto you those figures, befofe wee enter any
farther, to thinke wee make proceed alwaies in cer-
tentie, and knowe the thynges that wee intermedle
withall: so thei are the signes of all the arte, that fo-
loweth here to be taught.

And although there be many kindes of irrationall
numbers, yet those figures that serve in Cosike numbers,
bee the signes also of all irrationall numbers, and
therefore being ones well knowne, they serve in bothe
places commodiously.

These therefore be their signes, and significations
brieply touched: so their nature is partly declared be-
fofe.

f. Betokeneth number absolute: as if it had no
signe.

E. Signifieth the roote of any number.

F. Representeth a square number.

G. Representeth a Cubike number.

H. Is the signe of a square of squares, or Zenji-
zenlike.

I. Standeth for a Sursolide.

J. Doeth signifie a Zenji-cubike, or a square of
Cubes.

K. Doeth betoken a seconde Sursolide.

L. Doeth represent a square of squares squared
of Cube numbers.

I, oz a zenzizezike.

C, Signifieth a Cube of Cubes.

S, Cz, Crpyzelth a Square of Surfolides.

S, Cst, Betokeneth a thirde Surfolide.

C, S, S, S, Representeth a Square of Squared Cubes: oz a Zenzizezicubike.

G, S, Standeth for a fourthe Surfolide.

S, S, S, S, Is the sircne of a square of seconde Surfolides

G, S, S, Signifieth a Cube of Surfolides.

G, S, S, S, Betokeneth a Square of Squares, squarably squared.

E, S, S, Is the firste Surfolide.


E, S, S, Is the sircne Surfolide.

S, S, S, Dooth represente a square of squared Surfolides.

E, b, S, Standeth for a Cube of seconde Surfolides.

S, e, S, Is a square of thirde Surfolides.

g, S, Dooth betoken the seuenthe Surfolide.

S, S, S, S, Signifieth a square of squares; of squared Cubes.

And though I maie proccade infinitely in this sorte, yet I thinke it shall be a rare chance, that you shall neste this moche: and therefor this maie suffice. Notwithstanding, I will anon tell you, how you maie continuie these numbers, by progression, as farre as you like.

And farther you shall understand, that many men doe ruer more call square numbers zenzikes, as a shoter and aper name: other men call those squares the firste quantities: and the cubes they call seconde quantities; squares of squares they call thirde quantities, and Surfolides fourthe quantities. And so naming them all quanteties (excepte numbers and rootes) they do add to them for a difference; an onsmall name of number, as the do goe in order successively.
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As here followeth in example.

Firste.

Seconde.

Thirde.

Fourth.

Fifte.

Sicnte.

Seventhe.

Eigthe.

Nineth.

Tenthe.

Eleventhe.

Twelveth.

And so forthe, of as many as may be reckened.

But although sometimes accompte this the more ease waye: because the other names be com-
berous, yet those other names before, do expresse the quality of the number, better then these later names doe.

Scholar. I thank you double, if you are content to teach me double names: so shall I be ac-
quainted with both these names, as I shall chance on them in other menes bookes.

Therefore now you may proceed to numeration: whiche I thinkke it nerte.

Master. There be other. 2. Signes in often use, of whiche the firste is made thus—and betokeneth moze: the other is thus made—and betokeneth leste.

And where thet come in any number Cosike, or other, that number is called a compound number, be-
cause it consisteth of 2. numbers. And where neither of them is, the number is called uncompound, al-
though the signe be compound. For the compound signe, maketh not a compound number. And now I will proceed to numeration.
Of Cosike numbers.
Of Numeration in numbers
Cosike, uncompounde.

Master.
Numbers Cosike uncompounde, have no difficulty in their numeration: for ever more the number represented, so many of that Cosike denominatio be the numbers, rootes, squares, Cubes, squares of squares, or any other like, as there be unitics in that number.

So. 6. 9. 18. 6. numbers: And. 6. £. 18. 6. rootes: 2. 3. 18. 20. squares; 3. £. betokeneth. 3. £. Cubes.

Scholar. I see it well. For by this number, 20. 3. is not appointed any number absolute, of one certaine, but only so many quantities of that kind: which make bee. 8. 0. 16. 4. be one square. And 16. 9. be one square, then 20. squares make 180. And 16. 25. be one of those squares thereby represented, then. 20. squares make 500. And as for the signes, you taught me the before.

Of Addition.

Master.
This numeration is so plaine, that wee make passe from it unto addition: which is as easie also, if the quantities be of one denomination. For then needeth no more, but to adde the numbers together, and to put that same common Cosike denomination, to the total thereof.

Scholar. I take it thus, 20. £. added to. 30. £. will make. 50. £. And. 12. £. added to. 16. £. by sicheth for the. 28 £.

Master. As you doe easily see all the matter of this addition, so make you as easily conceiue, all the worke of substraction of substraction. For it is wrought as in vulgar numbers of like signes.

S. iy. Scholar.
The Arte

Scholar. Then if I abate 6. ℛ. out of 10. ℛ., there will reste 4. ℛ. And so 9. ℛ. out of 25. ℛ. doth leave 16. ℛ.

Master. This is all for numbers of like signes.

Scholar. What then if I would adde 10. ℛ. to 6. ℛ.: where the signes bee unlike: make it be done: sayng the: be not of one denomination: no signe Coſike.

Master. As well as shillynges make bee added with poundes, o: pence: and in like forme.

For the shall stand still as the: wer, with the signe of addition, which is this—|--: bartokeneth more.

So that 10. ℛ. put to 6. ℛ. maketh 6. ℛ.—|--:

10. ℛ. that is, 6. ℛ. more. 10. ℛ. 02. 6. ℛ. and 10. ℛ.

Scholar. And why not. 10. ℛ.—|--: 6. ℛ.?

Master. Because it is more orderly: to sette the greatest signe Coſike, so mo\thest in order.

As you saie. 2 o. shillynges, and 6. pence: rather then 6. pence and 2 o. shillynges.

Scholar. Then I se, if 15. ℛ. be added to 18. ℛ., it will make 18. ℛ. 15. ℛ. An so, 12. ℛ. torned with 2 0. ℛ., dooc make 2 0. ℛ. 12. ℛ. 12. ℛ.

Of Subtraction.

Master.

Subtraction is as easie: for it doeth depend onely of the signe of abatement, which is this:—|--: and signifieth leffe, 02 abatyng. And therefore if I would abate 6. ℛ., out of 10. ℛ. I must sette it thus 10. ℛ.—|--: 6. ℛ.: that is to saie, 10. ℛ. leffe 6. ℛ. 02 abatyng. 6. ℛ.

Scholar. Then if I haue 3 0 ℛ. and would abate out of the 12. ℛ. I must setit thus 3 0 ℛ.—|--: 12 ℛ. that is, 3 0. cuses saue 12. numbers. And if multiplication
of Coßike nombers.

tion and division, bee as easie, theı shall neade no
greate studie.

Of Multiplication.

Master.

Somewhat more laboure is there Multiplication in multiplication and division, to finde out the newe signes as I will tell you anon. But for finding of the nombers, the common multiplication and division doeth serve.

So that when, 12. 5. is multipliéd by, 2. it makeeth, 72. 2. And if, 24. 6. bee multiplied by, 5. 3. there riseth, 120. 3.

Scholar. This passeth my cunninge, for the finding of the newe signe: although the multiplication of the nombers, be as easie as can be.

Master. If you did well remembre, what you have learned before: the master would not seeme so hardye.

Doe not you knowe, that a roote multiplied by a roote, doeth make a square? And a square multiplied by his roote, doeth being for the a cube?

Scholar. That I knowe right well: and therefore a Square of Squares multiplied by his roote, will yeelde a Surfolide.

Master. Then by like reason, a Cube multiplied by a Square, shall make a Surfolide.

Scholar. In deede it is all one, to multiplie a cube by a Square, and a Square of Squares by a roote.

Master. Then for a generall rule, I will sette for the here a preadcente for you: whereby you maye knowe the newe signe, in all multiplication or division: not onely by sight very speedyly, but that you maye also commit it aptly to memorie.

Wherefore marke wel this table following: where you see in the higher rowe, a line of nombers, set in naturall
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natural progression: and under them you see the signs of cosmike numbers.

The table of Cosmise signes,
and their peculiar numbers.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>zc</td>
<td>8</td>
<td>cE</td>
<td>8'8'</td>
<td>8</td>
<td>8'c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>8'8'</td>
<td>8'8'</td>
<td>cE</td>
<td>cE</td>
<td>cE</td>
<td>cE</td>
<td>cE d/c</td>
</tr>
</tbody>
</table>

This table is largely set for the, in the title of progression, whereunto you may have recourse, if your number be to create for this table.

By this table may you easily knowe, the signe that shall serve for your newe somme, in multiplication.

As for example, if I dooe multiple squares by roote: I looke in the table, what numbers stande over them bothe, and putting those 2 numbers together, I take the toall in the same line, and under it I finde the newe denomination cosmise, which I should have

Scholar. I perceive over. zc, the number of 1 and over. zc, the number 2, which bothe added together make. 3. And because under. 3, I finde the figure or signe of. cE. I musst take that for the newe denomination.

Walker. You saie truthe.

Scholar. Then if I multiply. 12. zc, by. 8. cE, the somme will be. 96. cE cE. For over. cE, I finde 3 and over. zc, standeth. 6. which bothe together doe make. 9. and under. 9. I see. cE cE. which I take for the denominator.

And if the same rule be general, I am cunningge enoughe
of Cosike nombers.

Master. It is generally, for multiplication in this kinde.

Of Division.

At for division, you must abate the one number out of the other, to finde a newe denomination.

Therefore if you would divide 96, by 8, the quotiente will be 12, because that over the signe of your dividende, standeth 9. And over the division signe is set 3. Wherefore abating 3 from 9, there resteeth 6, under whiche is the signe, that I must take, to put to my quotiente.

Scholar. Then for an other triall, if I would divide 260, by 5, the quotiente will be 52. For, because that over of 5, I finde 17, and over of 5, standeth 5, then subracting 5, from 17, there resteeth 12 under whiche in the table I finde 3. So dividynge 20, by 4, the quotiente will be 5, and so of other.

Master. But and if you would divide 12, by 5, that must be set in forme of fraction, thus. \( \frac{12}{5} \).

So, 18, by 7, makes, \( \frac{12}{7} \), and 6, by 2, yeldeth \( \frac{6}{2} \), of whiche fractions, wee will speake amongst the fractions of Cosike compoide. For, the degenerate out of this kinde.

Wherefore this maie suffice brevily, for the customeable wooorkes of whole Cosike nombers.

Of Fractions in Cosike nombers.

And as for fractions, the woorkynge is like of fractions in numbers. Abstrate remembering onely that as those broken nombers, have a Cosike denomination annexed with them, so must
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that denomination followe the rules, now laste declared.

Wherefore I shall not neede to doe any more, but to set for the onely certain examples, of every kinde of woorkes in them.

Examples of Numeration.

\( \frac{1}{2} \mathcal{Z} \). Signifieth \( \frac{1}{2} \) of a Roote.

\( \frac{1}{3} \mathcal{Z} \). Betokeneth \( \frac{1}{3} \) of a Square.

\( \frac{1}{4} \mathcal{C} \). Representeth \( \frac{1}{4} \) of a Cube.

And so of all other soemes of Cosike signes: where by is intended, that the Cosike quantitie is divided into so many partes, as the denominatoz containeth, and there is here representeth onely so many of them, as the numeratoz doeth impute.

Scholar. Hereby I dooe perceive, that a fraction Cosike, maie significeth a number, and not onely a parte of an unitie, as it did in numbers Absoluto.

For when I saie \( \frac{1}{2} \mathcal{Z} \). if that Square be. 9, then that fraction significeth. 6. But if the Square be. 4, then that fraction doeth represente. \( 2 \frac{1}{2} \).

Likewies \( \frac{1}{4} \mathcal{C} \). if the Cube be. 8, then that fraction doeth significeth. 6. But if the Cube be. 27, then that fraction is equall to. \( 2 \frac{1}{3} \).

Master. You doe consider it well.

Of Addition.

Now for addition, take these examples.

\( \frac{1}{2} \mathcal{Z} \). added to \( \frac{3}{4} \mathcal{Z} \). doe make \( \frac{1}{2} \mathcal{Z} \). 02. \( 1 \mathcal{Z} \).

\( \frac{1}{4} \mathcal{C} \). joined with \( \frac{1}{2} \mathcal{C} \). doe make \( \frac{3}{4} \mathcal{C} \). 02 \( 1 \mathcal{C} \).

And in unlike signes.

\( \frac{1}{2} \mathcal{Z} \). added to \( \frac{3}{4} \mathcal{C} \). doe make

Thus by one common denominatoz.

\( \begin{array}{c|c}
16 & \mathcal{C} \\
15 & \mathcal{Z} \\
\hline
20 & \mathcal{C} \\
\end{array} \)
of Cośike nombers.

Of whiche I will speake more in the Binomials, and therefore will omitte it, till we come to them.

Scholar. As for the reste, I see it well: For the woorkke is all one with fractions Abstrakte.

And here the denominatión of Cośike signe is not varried, although here be used diverse multiplications.

Waller. And good reason: for the whole quotiente whiche is represented by that Cośike signe, is not multiplied, but certaine partes of it: and therefore oughte that Cośike signe, to stand unaltered, as the quantitie represented by it, is not multiplied nor altered.

Examples of Subtraction.

\( \frac{1}{2} \text{z}. \) abated out of \( \frac{1}{4} \text{z}. \) doe leaue \( \frac{1}{4} \text{z}. \)

\( \frac{1}{6} \text{z}. \) out of \( \frac{1}{7} \text{z}. \) there resteth \( \frac{1}{6} \text{z}. \)

\( \frac{2}{3} \text{z}. \) subtracted fró \( \frac{1}{4} \text{z}. \) doe leaue \( \frac{2}{3} \text{z}. \)

And in unlike signes.

\( \frac{1}{2} \text{z}. \) abated fró \( \frac{1}{2} \text{z}. \) doe leu \( \frac{1}{2} \text{z}. \)

\( \frac{1}{5} \text{z}. \) taken out of \( \frac{1}{2} \text{z}. \) the reste is \( \frac{3}{10} \text{z}. \)

Likewaies as in addition, so in this sorte of subtraction, there may be an other kinde of woorkke, whiche I will remit to the treatise of Binomiales.

Examples of Multiplication.

\( \frac{1}{3} \text{z}. \) multiplied by \( \frac{1}{2} \text{z}. \) doe make \( \frac{1}{6} \text{z}. \)

\( \frac{1}{3} \text{z}. \) multiplied by \( \frac{1}{3} \text{z}. \) hyngeth for the \( \frac{1}{9} \text{z}. \)

\( \frac{1}{5} \text{z}. \) multiplied by \( \frac{1}{3} \text{z}. \) doe yelde \( \frac{3}{15} \text{z}. \)

Here the signes doe alter, as in the multiplication of whole Cośike nombers.

E.g. Scholar.
The Arte

Scholar. This dooth somewhat trouble me: that the Cosike signes should change here, rather than in addition, or in subtraction: Seyng there was as moche multiplication, in any of them bothe, as there is here.

Mster. Hark the water well, and you shall be some satisfied.

For in addition and subtraction, the multiplicatio serveth onely for the reduction of the 2 fractions, unto one denomination: And therefore in them, you neuer multiply the numerators together: but you multiple crosse waies, the numerators of the one, by the denominators of the other, where as in multiplication, you use no reduction, but doe make a plain multiplication.

And so likewaies in diviision, there is used no means of reduction: and therefore in it the signes must alter, as before is declared.

Examples of Division.

\[ \frac{\frac{1}{7} \frac{1}{3}}{\frac{1}{4} \frac{1}{5}} \] divided by \( \frac{\frac{1}{6} \frac{1}{7}}{\frac{1}{8} \frac{1}{9}} \), doe make in the quotiente

\[ \frac{\frac{1}{7} \frac{1}{3} \frac{1}{4} \frac{1}{5}}{\frac{1}{6} \frac{1}{7} \frac{1}{8} \frac{1}{9}} \]

\( \frac{\frac{1}{7}}{\frac{1}{8}} \) divided by \( \frac{\frac{1}{9}}{\frac{1}{10}} \), doeth peld \( \frac{10}{9} \). Or els \( \frac{10}{9} \),

For seyng I shall divide. \( \frac{1}{8} \) by \( \frac{1}{9} \), I must therefore subtrah 3 from 3, and so refteth nothing, whiche is signified by this Cipher. 0, and that standeth over the signe of number: therefore the fraction, that is as the quotiente, must be taken as a number Absurdtie.

Likewise \( \frac{\frac{1}{7} \frac{1}{3}}{\frac{1}{4} \frac{1}{5}} \), divided by \( \frac{\frac{1}{6} \frac{1}{7}}{\frac{1}{8} \frac{1}{9}} \), doeth make \( \frac{10}{9} \), that is to saye 3. And so \( \frac{\frac{1}{10} \frac{1}{7}}{\frac{1}{8} \frac{1}{9}} \), divided by \( \frac{\frac{1}{10} \frac{1}{7}}{\frac{1}{8} \frac{1}{9}} \), doeth bying of the \( \frac{10}{9} \) \( \frac{1}{8} \),

Scholar. This is sufficient for division. Now if you thinke good to speake of progression, I can not but remember you of your promise.
of CoSike nómbers.
Of Reduction.

Máster.

Although Reduction should go in order be Reduction, for Progression, yet seeing this Reduction, consisteth in the onely numbers, and not in the signes: and therefore agreeith with vulgar reduction of fractions (as here you maie see before in diverse examples) therefore will we omitte it, and go in hande with Progression: which is moze strange.

Scholar. I praise you so: For I see this reduction, is but to reduce the greater fraction, to a letter in nóber: as I learned long a gone by your other booke.

Of Progression in CoSike signes.

Máster.

Progression is thus wroughte: First sette bounde as many vulgar nóbers, in their naturall progression, as you liste to have CoSike signes, that by them you maie the better know, the true places of the CoSike signes: so that you set in the first place a Cipher, and under it. And then under, 1. set. 2. under. 2. put. 3. and under. 3. write. As you see in the table followinge. And by these shall you set, as many as you liste.

For all the vulgar nóbers, which you haue set in the higher rowe, be other compounde nóbers, or else uncompounde: and if the place, where you would set any CoSike signe, be noted with a nóber uncompounde, then must there be set one of the Surfolides.

For under the first nóber uncompounde, you must set the first Surfolide, and the second under the second nóber uncompounde: and the thirde under the thirde.
The Arte

and so for the.

The numbers uncomposite are these in their progression.

5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, etc.

Under aethe, 5, must you set 5/z, and under 7, 5/z, and under 11, 5/z, and so for the, till you come to 67, under which you must set 5/z, and under 71, you must set 5/z, and so as far as you list.

But for any other place, because the vulgar number is compound, that is set as the peculiar number, in the higher rewe therefore the cosine must needs be compound, other as 2, 02, or 3, 02, etc. of both. And if it be compound of 2, then set dounce, 5, so often tympes, as 2 is in the composition of that number.

As for example: 16, is compound of 2, twelve tympes (not by addition, but by multiplication, as in saying, twice, twelve tympes, twice.

Scholar. I perceive twice, 2, to bee, 4, and twice that to be, 8, and twice that to make, 16.

Master. So make you workke backwater, in saying, 16, divided by 2, maketh, 8, that is ones: then, 8, by 2, maketh, 4, that is twise. Again, 4, by 2, maketh 2, that is thirse: and, 2, for himself, is the fourth: wherefore under, 16, I must set dounce, 5/z, 5/z.

And so under, 32, I must sette, 5/z, in one thus, 5/z, 5/z, 5/z.

And under, 64, I shall sette it, 6, tympes, thus, 5/z, 5/z, 5/z, 5/z. Because, 64, is made of, 6, multiplications by 2.

Scholar. Here by I see, that under, 5, I must put 3, tympes that signe: and under, 4, twise the same.

Master. So must you in deed.

And now for other places, if their numbers bee compound
of Cosike numbers.

pounde of 3. onely, then must you set doune the signe of Cube, as oftentymes as 3. is multiplied, to make that number.

As for example. 27 is compounde onely of 3. and not of 2. (for of all other compounde numbers herein then of soche as be copounde of 2. o2. 3. we take no regard.) And 3. multiplied thuse, doeth make 27. in sayling 3. tymes 3. thuse. And therefore under 27. I shall set this signe of 3. three times, thus 3 3 3. whiche betokeneth a Cube of Cubes Cubike.

But if the number bee compounde, bothe of 3. and 2. then for every tyme that 2. is multiplied, to that composition, I shall sette 2. and for every tyme that 3. is multiplied, I shall set 3. remembryng full to set 3. befoxe 2. and not after hym.

As for example. Under 24. I shall set 3. 3. 3. because that 2. 2. 2. 3. that is to saye 2. tymes 2. twicethuse doeth make 2 4. 2, by resolution, thus 2 4. divided by 2, yieldeth 12. For that firste 2. set 2. Again 12. divided by 2, yeldeth 6; for this seconde 2. set 2. also. Then divide 6. by 2, and it maketh 3. For the 2. I must set 3. and for 3. I must put 3. and so all to gether maketh 3. 3. 3. 3. in the 2 4. place.

Likewaies under 36. I must sette 3. 3. 3. 3. because that 2. 2. 2. 3. doeth make it, that is 2. tymes 2. thuse thuse. And by resolution, thus 36. divided by 2. yeldeth 18. For that 2. I set 2. Again 18. divided by 2. maketh 9. For that 2. I sette doune againe 9. Thirdly, for because 9. can not bee divided by 2. but by 3. 3. tymes: therefore I must sette doune for those twoo. 3. thuse 3. 3. 3. so the whole signe is 3. 3. 3. 3.

Now if the number of the place, or pectileare number, bee compounde of one of theim twoo, with some other number uncopounde, then must we toynge their signes together.

As 1 0. is compounde of 2. and 5. therefore must 1 0.
The Aarte

let bnder. I o. the signe that is in the fift place, whic
che is $\frac{z}{2}$, and before it I muste set the signe of $\frac{z}{2}$. so
2. So must that signe be $\frac{z}{2}$. 2.

Likewaies, because $\frac{z}{2}$ is compound of $\frac{z}{2}$ and $\frac{z}{2}$
I shallione together the signe of $\frac{z}{2}$, and of $\frac{z}{2}$, and
make it. $\frac{z}{2}$.

Scholar. So I understande it now, that I cannot
misse it. Saue that for lacke of bse, and throughe for
getfulness, when I heare the name of composition
in numbers, I doe mistake it sometimes for addition,
els here can be no errore. For when I doe consider,
that $\frac{z}{2}$ is compound of $\frac{z}{2}$, $\frac{z}{2}$, that is twise $\frac{z}{2}$, and $\frac{z}{2}$
(sith $\frac{z}{2}$ tymes 2. maketh 4; and $\frac{z}{2}$ tymes 4. maketh
$\frac{z}{2}$.) I maie aone consider, to set $\frac{z}{2}$, twise before $\frac{z}{2}$.
and then it will be $\frac{z}{2}$. $\frac{z}{2}$. to be put in the 20. place.

Likewaies in the 21. place, I set $\frac{z}{2}$, seyng 21
is compound of $\frac{z}{2}$ and $\frac{z}{2}$, and $\frac{z}{2}$. is the signe to the
third place, as $\frac{z}{2}$ serveth for the 7. place.

Master. What shall you set in the 84. place?

Scholar. 84. is compound of 2.2.3.7. therefore
his signe must be $\frac{z}{2}$ $\frac{z}{2}$. $\frac{z}{2}$.

Master. Now I see, you are cunning enough in
this, and therefore take here this table, for a patronc;
and then will we proceed to the worke of Cossike nom-
bers compounde.

The table for progression Cossike,
which maie increase it self infinitely,
without any difficultie.
In this table, \( z \), \( z \), and \( z \) are the groundes; of all the reste above them. For of these thse, all those other bee made.

\[ z, z, z \]
The Arte
Of Cosike numbers compound.

Cosike numbers compound are made by addition of two more simple Cosike numbers together:

As. 6. z. — — 5. z. 0.2.
12. z. 4. z. — — — 3. 9. and so for the in diverse forms, which be infinite. Howbeit for brevity, we make comprehended under the same name (because of the like work) all other residualles Cosike, which be made by subtraction: as. 3. z. — — 4. z. And also those that bee made by addition and subtraction, bothe together: As. 9. z. — — 4. z. — — 6. z.

In whose numeration is no hardnes.

Scholar. Then your rules maie be the shorster.

Of Numeration.

Master.

His Numeration is easily understood by addition of simple Cosikes. For this is the forme. 6. z. — — 10. 9.
that is. 6. Squares, move. 10. nöbers.
Likewises. 8. z. — — 11. z. is
8. Cubes and. 11. z.

Now for residualles, take these examples. 9. z. — — 12. z, which is. 9. Squares of Squares, saue. 12. Cubes. Also. 4. z. — — 15. z.
that is. 4. sursolides, abatyng. 15. Squares.

And for bothe together, this is the forme.
10. z. — — 6. z. — — 30. z. which signifies

Scholar. This is plaine. For so maie I understand of all other As. 9. z. — — 3. z. — — 8. 9

Master.
of Cossike nombers.

Master. It were more orderly to keep the signes of more and lesse in order, then to follow the order of the cossike signes: because that addition, is orderly placed before subtraction. So were it better to set them thus 9.5-2. | - 8.9. | - 5.8. Hence it appeareth all is one in these kindes of numbers, but not so in other Surden nombers, where the order followeth of necessity, as shall be declared in their place more largely.

Of Addition.

In addition, you must have consideration of the cossike signes: for noe other number, may bee added into one, then may be appertain to one signe cossike.

As in vulgar denominations, you doe not add the numbers of shillings to the numbers of pence: but you take shillings to shillings, and pence to pence: 

So in cossike nombers, Cubes muste bee joined to Cubes, Squares to Squares, and generally, like to like.

Scholar. If this be al, I can marke it well enough.

Master. There is somewhat more to be considered, that if there bee any signe in the one number, which is not in the other, that seuerall signe with his number, must bee sette doun: with his figure of .02. as it standeth there.

And farther, touching those twoo signes, which bee the figures of more and lesse, you must give regard, whether thei bee like or unlike, in those numbers that must be added: For if thei bee like in numbers, of one denomination, then must thei so remain as thei be. But if thei bee unlike, censure abate the smaller number of them, that followe those as if
The Arte

Unlike signes, out of the greater: and sette doune the reste, with the signe of the greater number.

Scholar. By examples, I shall better conceive those rules.

Master. Take these examples.

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<tr>
<th>10.3</th>
<th>12.9</th>
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<tbody>
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Here haue I varied one example diversly, to the utter you make marke the use of your rules in them. And so the reason of those rules, you shall marke those
those examples well.

For where in the firste example, bothe the signes are — — , it must needes be, that after the addition of the firste numbers, the seconde must be added with the signe — — .

In the seconde example, where bothe the signes be — — , because there wanteth. 21. 9. of the first. 10. 9.

Therefore is it reason, that bothe those wantes should be sette downe with the signe of — — ; and so in the thirde and fourthe examples.

In the fifth example, the seconde somme is not fully. 4. 9. but there wanteth of it. 8. 9. and therefore if you put downe the. 4. 9. fully, you must abate. 8. out of the. 12. 9. in the higher somme: and so of the other examples.

But for more praetise, and better declaration of the use of them, here are other examples, of more variety.

\[
\begin{align*}
20. 3. & \rightarrow 9. 3. \rightarrow 120. 0. \\
15. 3. & \rightarrow 5. 3. \rightarrow 16. 0. \\
35. 3. & \rightarrow 14. 3. \rightarrow 104. 0.
\end{align*}
\]

\[
\begin{align*}
16. 9. & \rightarrow 28. 9. \rightarrow 16. 9. \\
12. 9. & \rightarrow 12. 9. \rightarrow 19. 9. \\
\end{align*}
\]

In the firste example of these. 2. you see. 120. 0. With the signe of lefte, to be added with. 16. 0. With the signe of more: and therefore, seying the signes of one Coسike denomination disagree, I doe subtracte the lesser, out of the greater: and that. 104. Whiche remaineth, I doe set downe with the signe of ٣. isi. lefte
The Arte

leste, because the remainder is of that number, that
bare that signe.

And in the second example, the placing of the signe
— before — maketh numbers to bee sette be-
fore squares: and so the like denominations, doe not
stande one over another, yet is the woode dooen as
if they did stande eche over his like.

Scholar. I praye you lette me trie my cunninge,
with an example of twoo.

\[
\begin{align*}
17 \cdot \frac{3}{2} \cdot \frac{3}{4} & = 10 \cdot \ell & = 2 \cdot \ell \\
16 \cdot \frac{3}{2} \cdot \frac{3}{2} & = 12 \cdot \ell & = 6 \cdot \ell \\
16 \cdot \frac{3}{2} \cdot \frac{3}{4} & = 17 \cdot \frac{3}{2} \cdot \frac{3}{4} & = 10 \cdot \ell \\
& = 12 \cdot \ell & = 8 \cdot \ell
\end{align*}
\]

I set the example, as numbers came to my mynde:
but I had almost set my self on grounde: saue that I
called to remembrance, the comparison that you
made, to vulgar denominations of poundes, shilling:
ges, and pennies: and so was instructed to place-every
several denomination severally. And to sete the
greatest denomination first, & eche other in his order.

Now will I proue another example, of twoo.

\[
\begin{align*}
3 \cdot \frac{3}{4} & = 4 \cdot \ell & = 20 \cdot \ell \\
20 \cdot \ell & = 8 \cdot \frac{3}{4} & = 16 \cdot \ell \\
3 \cdot \frac{3}{4} & = 24 \cdot \ell & = 8 \cdot \frac{3}{4} & = 36 \cdot \ell
\end{align*}
\]

\[
\begin{align*}
13 \cdot \ell & = 8 \cdot \ell & = 4 \cdot \ell \\
7 \cdot \ell & = 6 \cdot \ell & = 7 \cdot \ell \\
20 \cdot \ell & = 2 \cdot \ell & = 4 \cdot \ell & = 7 \cdot \ell & = 6 \cdot \ell
\end{align*}
\]
of Co$\mathit{b}$ike nombers.

\begin{align*}
6 \cdot 8 & \quad - \quad 10 \cdot 2 \quad - \quad 8 \cdot 9. \\
4 \cdot 8 & \quad - \quad 17 \cdot 9 \quad - \quad 7 \cdot 2 \quad - \quad \\
10 & \quad - \quad .3 \cdot 2 \quad - \quad .9 \cdot 9.
\end{align*}

\begin{align*}
4 \cdot 8 \cdot 2 & \quad - \quad 5 \cdot 8 \quad - \quad 6 \cdot 2. \\
8 \cdot 2 & \quad - \quad 8 \cdot 2 \quad - \quad 10 \cdot 2. \\
4 \cdot 8 \cdot 2 & \quad - \quad 8 \cdot 2 \quad - \quad 8 \cdot 2 \quad - \quad 4 \cdot 2.
\end{align*}

Master. You have done well: And for proofe of your woork, you maie in this arte not onlye proue it, by the contrary kynde, as you did in nombers Abstraite, but also by the resolution of all those Co$\mathit{b}$ike nombers into nombers Abstraite, takynge any nomber for a roote and then the Squares and Cubes, &c. accordingly. As her in this table, you maie hastily see, but more largely in the table at the ende of nombers figuralie.

\textit{A table for trialle by resolution, of any woork in this arte.}

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
$\mathbb{2}$ & $\mathbb{4}$ & $\mathbb{8}$ & $\mathbb{16}$ & $\mathbb{32}$ & $\mathbb{64}$ & $\mathbb{128}$ \\
\hline
$\mathbb{3}$ & $\mathbb{9}$ & $\mathbb{27}$ & $\mathbb{81}$ & $\mathbb{243}$ & $\mathbb{729}$ & $\mathbb{2187}$ \\
\hline
$\mathbb{4}$ & $\mathbb{16}$ & $\mathbb{64}$ & $\mathbb{256}$ & $\mathbb{1024}$ & $\mathbb{4096}$ & $\mathbb{16384}$ \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline
$\mathbb{256}$ & $\mathbb{121}$ & $\mathbb{1024}$ \\
\hline
$\mathbb{6561}$ & $\mathbb{19683}$ & $\mathbb{59049}$ \\
\hline
$\mathbb{65536}$ & $\mathbb{262144}$ & $\mathbb{1048576}$ \\
\hline
\end{tabular}

And if this table in any parte, seeme to shooze or to little:
The Arte

little: you may have recourse to the table, at the end of figuralle numbers, which therefore is made large and generall: so that it may well be called the rude: full table, or table of ease.

But now for triall of the laste example: first there is $4 \times 2 = 8$ whose roote I take $2$ and therefore those $4 \times 2 = 8$ make $256$. which I lette oone in number Abstrate.

Nexte is $5$ squares, whiche according to that roote, must nedes be $20$ and that $20$. I lette oone also: and then $6$ rootes, whiche make $12$. And all thei yeide $288$ and that is all the firste somme.

Then for the seconde somme, I see firste $8$. Cubes, whiche make $64$ to bee added. Then foloweth $8$ squares lesse, that is $32$ to bee abated, and also $6$ rootes lesse, that is $20$ also to bee abated: So must I abate $52$ (for theim both) out of $64$, and then there remayneth but $12$: whiche added unto $288$, of the firste somme $300$. doe yeide $300$.

Now if the totall agree with this, then is the woozke good.

For triall whereof, I resolue $4 \times 2 = 8$ into number Abstrate, and then will make $256$. Then $8 \times 2 = 16$, whiche bothe yeide $320$. Then foloweth in the same somme $3 \times 2 = 64$ and $4 \times 2 = 8$, to be abated. The $3 \times 2 = 64$ and the $4$ rootes yeide $8$, whiche together do amounte to $20$, and that must bee abated fro thesai'd somme of $320$ and then there remaineth onely $300$ agreeable to the former somme aboue the line.

Scholar. This proose I like well: And I perceive that if I would woozke the like, taking for the roote $3$, or any other number, the proose will sucede a like.

Master. How to make an eande of Addition, be-
of Cosike nombers.

cause you shal the better remembre the rules of it, I will giue you them in this brieve forme.

In greatenesse like and signes also,
Add like to like there nedes no moe:
And where the greatenesse disagree,
Place eche by other sufferally.

with signe of eche, as doeth require,
But if the signes unlike appeare:
Then from the more abate the lesse,
The greater his signe with the excesse.
will make the somme,
Of that addition.

The proosfe is by resoluyng,
Eche nomber into his reckenyng.

This lesson doeth containe the former rules onely
in brieve, and therefore needeth no declaration: but the
greatenesse doeth betoken the Cosike denomination,
and signes betoken specially, and the
signes of more and lesse, and no other signes.

Scholar. This brieve lesson will helpe memorie moche; and shall sufficce for the rules of Addition.

Of Subtraction.

Mster.

Then for subtraction, this shall you
marke in especiall: that when your
nombers are fette done, after the
common manner, strike the total, and 1. Rule.
than the deducution: you shall consi-
der well, whether the signes bee
92. For in the de-
ducution, if you have—then must that be subtrac-
ted from the like aboue.

And if that somme in the deducution, that hath the 2. Rule.

X. 3. signe
signe —— bee greater then the number of the like quantitie ouer hym, with the like signe —— then abate the higher out of the lougher, and write the reste with this signe ——.

3. Rule.

But if the like quantitie in the totall, have the signe ——, then add both the numbers together and set them under the line with that signe ——.

4. Rule.

And if the seconde somme (that is the deduction or abatement) with any number, have this signe of lesse ——, it must be accounted for more, and must be added to the like number over it, excepte the over number have the signe of lesse also: For then must you abate the lesser, out of the greater, and sette doun the reste, with the signe of the greater number: which the have at this conference: I mean to reardc what the signe of the seconde somme is by estimation, and not by writing, for they are contrary.

Scholar. I see good reason in this: For in any abatement, the more is abated, the lesse by so moche shall remain: and the lesse is abated, the more doeth remain by so moche.

5. Rule.

Master. Yet one thyng more is to bee marked, that if there be some denominations, in the one some that are not in the other, you shall mark in which somme thei bee. For if thei bee in the firste, then shall thei kepe still their owne signe. And if thei bee in the seconde somme, which is the deduction, then shall thei change their signe to the contrary: But where ofuer thei be, thei must be set in the remainder.

Scholar. I can better understande you, then remembre those rules.

Master. Then take this brief lesson, after to bee remembred, then to bee understande, but by the letter before, and by the examples folowyng. But memoire likewise well loche aide.
of Cosines of Numbers.

A brief rule of Subtraction.

1. When signes and greatness both be agree,
   Your worke proceedeth for the commonly.

2. But if the batement be greater bee,
   The excess shall change his signe thereby.

3. And where the signes doe disagree,
   The higher signe must rest dely:
   And though the batement be the greater,
   The rest still ioyneth both sommes together.

4. If quantities doe disagree,
   Place them with signes all severally:
   The etall kepeth the signe be bad,
   The batementesfyll, to change is glad.

Scholar. Now some examples, will lighten these rules well.

Master. I will propounde the like, as I did in addition, to the intete you make: judge the likenesse, and diversities in bothe woorkes.

| 10.5 | - | 12.9 |
| 4.5 | - | 8.9 |
| 6.5 | - | 4.9 |

| 10.5 | - | 8.9 |
| 4.5 | - | 12.9 |
| 6.5 | - | 4 |

\[ \text{Ex.} 10.5 \]
The Arte

10. £. —— 12. £.  
4. £. —— 8. £.  
6. £. —— 20. £.  

10. £. —— 12. £.  
4. £. —— 8. £.  
6. £. —— 20. £.  

The firste and thirde examples be very plaine: and in the seconde where 12. should bee abated out of 8. there is 4. to sewe: and therefore I abate the higher, out of the lougher, and I sette downe. 4. with the signe of wantying, or abatement.

In the fouther example: because the higher number is the lesse, I doe subtrahc him out of the nether, and sette downe the reste. 4. with a contrary signe of ——.

But in the 4. later examples, where the signes do disagree, the numbers that followe the signes, are not substracted one from an other, but are added together: and they take still the higher signe. Because in value, the signe of abatemente is contrary, to that it appeareth to bee.

And for your exercise, to make you full prompte in this arte, I have set for the more examples.

6. ℛ. —— 120. ℛ.  
8. ℛ. ℛ.  
9. ℛ. —— 40. ℛ.  
9. ℛ. ℛ. —— 89. ℛ.  

160. ℛ. —— 3. ℛ.  
89. ℛ. —— 1. ℛ. ℛ.  

3. ℛ.
of Cosike nombers.

\[
\begin{array}{ccc}
3 \cdot z \cdot & + & 18 \cdot z \cdot \\
12 \cdot z \cdot & - & 3 \cdot z \cdot \\
6 \cdot z \cdot & - & 6 \cdot z \cdot \\
\hline
18 \cdot z \cdot & - & 3 \cdot z \cdot \\
12 \cdot z \cdot & - & 3 \cdot z \cdot \\
6 \cdot z \cdot & + & 6 \cdot z \cdot \\
\hline
3 \cdot z \cdot & + & 18 \cdot z \cdot \\
12 \cdot z \cdot & - & 8 \cdot z \cdot \\
\hline
3 \cdot z \cdot & + & 6 \cdot z \cdot \\
\hline
4 \cdot f \cdot & + & 10 \cdot c \cdot \\
5 \cdot f \cdot & - & 12 \cdot f \cdot \\
\hline
4 \cdot f \cdot & + & 10 \cdot c \cdot \\
5 \cdot f \cdot & + & 18 \cdot f \cdot \\
\hline
\end{array}
\]

Here in the firste example, where I would abate 9 \( c \) out of 6 \( c \). I maie easilie perceive, that there are 3 \( c \) to solwe. And therefore doe I sette donue, 3 \( c \). with this signe --, which signifieth wante of a batemente: and the 2. nombers that followe the vnlike signes, I set donue both the added into one: and put thereto the signe of the totall of 2 outermoke somme.

In the seconde example, there is the like woork: For in abating 9. out of 8. I finde, 1. to solwe: that 1. doe I set donue with his denomination of 5 \( s \). and the signe. --.

And the number 89 that followeth the signe --. In the seconde somme, standeth in forse as --, for the lette is abated, the more must remain: therefore in the remainder, I set not the signe of more, before that number of 89. but I put it in the firste place of the somme: which place of it seif, signifieth still more.

\[x \cdot i \cdot y.\]

And
The Aarte

And because over that number 89, there are no numbers in the total, therefore I muste putte downe that somme as it is, without addyng to it, or abatynge fro it, in it self.

Scholar. Those 2. examples might be set thus, as I thinke, because the places doe so require.

\[
\begin{align*}
6.9 & + 120.9 \\
9.9 & = 40.9 \\
\hline
\quad & = 160.9
\end{align*}
\]

\[
\begin{align*}
8.9 & + 160.9 \\
9.9 & = 8.9 \\
\hline
\quad & = 8.9
\end{align*}
\]

Master. Remember your self well, and mark the remainder how it is written.

Scholar. I see my owne oversight: For no number maie begin, with signe of lesse: and therefore must their places be altered of necessity, and set in order as they were before.

Master. Then for all the rest of the examples, or any other like, I shall not neede to give you any farther instruction: Sith that by these former, you maie judge of all other.

Proofo. And for the examinacon of your worke, the trialle by resolution doeth serue here, as well as els where: remembrynge onely (as the order of subtraction maie admonishe you) that the somme of the totalle, whiche is the firste somme, must counteruale the other bothe sommes: that is of the deduction, and of the remainder.

So to trie the firste example, takynge 3. for a roote:

\[
\begin{align*}
6.9 & = 162. whiche I put to 120, and it yeldeth 282. Then in the seconde somme 9.9 are 243. Wherof 40. must bee abated for the signe ——, so is
of Cosike nombers.

is that somce, 203. Again in the remaine, 3, c2, are 81, whiche must bee abated out of, 160, and so resteth 79, whiche with, 203, doe make, 282, agreable with the firste somme.

Scholar. This doe I well understande, and prai(te you to procede to multiplication.

Of Multiplication.

Master.

In multiplication, there is no difficulthe, so that you doe well marke the signes — and —, whiche beyng bothe like, will haue the signe — sette in the totall: and beyng unlike, they will haue in the totall the signe —.

And likewaies in division — — divided by —
02 contrary waies — — by — — will alwaies have in the totall — — —: but — — divided by — — 02 — — —, will make alwaies — —.

Whiche rule for ready remembraunce, I have giuen you here in meter.

Who that will multiply,
Or yet divide trulie:
Shall like still to haue more,
And mislike lese in store.
Their quantities doe kepe suche rate,
That, M. doeth add: and, D. abate.

Scholar. So meane you, that like signes multiplied together, doe make more, 02 — —: And unlike signes multiplied together, doe yelde lesse, 02 — —.

Master. So is the rule. But to go forward now: of the nexe difficulthe, as touching Cosike quantities that chaunge their denomination, here is no more to bee
The Arte

bee saide, then was taught in multiplication of numbers Cosike uncompounde, and in the table set for the for the chauge of their names.

Scholar. I understand, that in multiplication (that is. P.) their figures must bee added. And in. D. (or division) thei muste bee abated. Therefore a fewe examples shall suffice for the reste.

Master. Take these for a presidente, of all that woorke:by whiche you make judge of all other like.

\[
\begin{align*}
10 \cdot \text{c} &\quad -\quad 9 \cdot \text{z} &\quad -\quad &\quad 20 \cdot \text{c} \\
5 \cdot \text{z} &\quad -\quad &\quad 7 \cdot \text{z} &\quad -\quad &\quad 8 \cdot \text{z} \\
\hline
80 \cdot \text{c} &\quad -\quad &\quad 72 \cdot \text{z} &\quad -\quad &\quad 160 \cdot \text{c} \\
70 \cdot \text{z} &\quad -\quad &\quad 63 \cdot \text{c} &\quad -\quad &\quad 140 \cdot \text{z} \\
50 \cdot \text{z} &\quad +\quad &\quad 45 \cdot \text{z} &\quad +\quad &\quad 100 \cdot \text{c} \\
50 \cdot \text{z} &\quad +\quad &\quad 115 \cdot \text{z} &\quad +\quad &\quad 83 \cdot \text{c} &\quad +\quad &\quad 68 \cdot \text{z} &\quad +\quad &\quad 160 \cdot \text{c}
\end{align*}
\]

\[
\begin{align*}
15 \cdot \text{z} &\quad -\quad &\quad 12 \cdot \text{z} \\
14 \cdot \text{z} &\quad -\quad &\quad 2 \cdot \text{z} &\quad -\quad &\quad 5 \cdot \text{z} \\
\hline
75 \cdot \text{z} &\quad -\quad &\quad 60 \cdot \text{c} \\
30 \cdot \text{b/z} &\quad +\quad &\quad 24 \cdot \text{c} \\
210 \cdot \text{z} &\quad -\quad &\quad 168 \cdot \text{z} \\
210 \cdot \text{z} &\quad -\quad &\quad 30 \cdot \text{b/z} &\quad -\quad &\quad 75 \cdot \text{c} &\quad -\quad &\quad 168 \cdot \text{z} \\
\hline
24 \cdot \text{c} &\quad -\quad &\quad 60 \cdot \text{z}.
\end{align*}
\]

Scholar. I perceive, that these woorke doe appere more hard, then thei bee in deede, and that because of their straunge formes: but by des I trulle to bee acquainted with them well enouff, and therefore I will begin with more easie examples. As these bee, that folowe
of Cozike numbers.

followe here.

\[
\begin{align*}
18. \ell & \rightarrow 20 \ell \\
15. \ell & \rightarrow 4 \ell \\
\hline
72. \ell & \rightarrow 80 \ell \\
270. \ell & \rightarrow 300 \ell \\
\hline
270. \ell & \rightarrow 300 \ell \rightarrow 72 \ell \rightarrow 80 \ell.
\end{align*}
\]

\[
\begin{align*}
16. \ell & \rightarrow 14 \ell \\
8. \ell & \rightarrow 7 \ell \\
\hline
112. \ell & \rightarrow 98 \ell \\
128. \ell & \rightarrow 112 \ell \\
\hline
128. \ell & \rightarrow 112 \ell \rightarrow 112 \ell \rightarrow 98 \ell.
\end{align*}
\]

And this I see farther now, that these woorkses seeme more difficulte to looke on, then that be in practise, if a manne give good bode to the signes, and the quantitics.

Master. Before we go any farther, I will shewe you somewhat of the reason, why the signes ought to chaunge. And that by twoe plaine woorkses, in numbers Abstraitte. As here foloweth.

Where you see, that when

<table>
<thead>
<tr>
<th>I had multiplied</th>
<th>16</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>by 20 it made</td>
<td>320</td>
<td>240</td>
</tr>
<tr>
<td>that is in all, 560</td>
<td>64</td>
<td>48</td>
</tr>
</tbody>
</table>

But because the multis are ought not to be so moche

<table>
<thead>
<tr>
<th>that is, 448</th>
</tr>
</thead>
</table>

I shall multiply the higher somne by 4, and abate that out of the former total.

V.4. Whiche
The Arte

Whiche thing you see here done by, ——. 64.
——. 48, whiche bothe make, 112, to bee deduced
out of 360, and so remaineth 448. The 1st fumme
that commeth of that multiplication.

Scholar. This I understaunde well: and
maie proye it in this sorte. 16, ——. 12.
maiaeth 28: and 20, ——. 4, is 16.
Then if I multipie 28 by 16, it wil yeale
448, as the wooyke here declareth.

And hereby maie I judge, of Cosike num-
bors like these.

Master. Yet one example more will I propound
because I would put you out of all doubte. Wherfore
marke this soyme of wooyke.

Here you maie see, that if
the 1st fumme of 24 ——. 3
wer multiplied by 15 it would ——. 48 ——. 6.
make, 360, ——. 45, that is
315. But it ought not to bee so ——. 366 ——. 93.
moche, but lese by 2 tymes
24 ——. 3, that is, 48 ——. 6:
because the multiplier doeth wante 2 of 15.

And so abatinge, 42, 02, 48, ——. 6, out of 315.
there resteth, 273, whiche is the 1st fumme total, when 21
is multiplied by 13, wherby the multiplication is de-
clared to bee good.

And so, because that —— multiplied with
— doeth make ——: marke here, that you maie not a-
date fully, 48, but 48 ——. 6.

Then seeing in abatemente, the signes in figure
are contrary to their owne estimation and force: ther-
fore that, 48, must be made —— and the —— be-
foze, 6, tourned into ——.

Scholar. I see it well, it must needs be so.

For if thei were set, to bee subtracted, then should
thei stande so, 48, ——. 6, whiche declareth that 42
should
of Cosike nombers.

Should bee abated.

But when the same nombers, are set in onggeste othet to be added: as it is here in working of multiplication, then must thei be written thus. —48. 6. declaring that if you abate, 48, you must add 6. again, because you abated 6, more then you ought.

Master. You understand it well. Wherfore here will wee make an ende of multiplication: sith there reseth nothing but the proofe of it: where the same bee wrought by resolution, of all the Cosike nombers, into nombers abstrakte, as in other kindes before. Once considerying that the resolutions of the first and seconda sommes, must be added together.

And therefore if youliste to prove the firste example taking 2 for the roote, you shall finde the firste sume 80. —36. —40. that is 156. And the seconde somme is 20. —14. —8. that is 26. The thirde somme is 1600. —1840. —664. —272. —320. whiche make 4056. And so doeth. 156, multiplied by 26.

Scholar. This make I prove at any tyme: so that you shall not neede to state aboute it.

Of Division.

Master.

Division is nexte in oser, and agreable in the generall rules: and hath nor more speciall, than the very nature of the woode dooth require.

For as concerning the signes of — and ——, the same order is here, as is in multiplication. And touching the Cosike signes, it is all one with that I stated in division of nombers Cosike uncompounde.

Scholar. Then if we examples make supplie the
The Arte

Declaration of the use of the rules, with the practice thereof.

Master. Take these for your purpose.

An example of the first work.

\[ \begin{align*}
& 60. \\
& \text{The removing of the distill,} \\
& \text{for the second work.} \\
& \text{The proofe in numbers Abstracte,} \\
& \text{accompanying. 2 of roots.} \\
& 3 \quad 480. \\
& 192. \quad \text{320.} \\
& 24. \quad \text{16.} \\
& 480 \quad \text{320.} \\
& \text{Here I have not only parted} \\
& \text{the worke, for your ease in understanding: but I have also put} \\
& \text{against it, the declaration of the same, by resolving the Cossike} \\
& \text{numbers, into numbers Abstracte.} \\
& \text{And finally, I have putte one example of the same numbers,}
\end{align*} \]
of Cosike nombers.

nombers, after the vulgare fo:me: all whiche. 3 agree together: and vouche one an other.

Scholar. Yet I piae you woooke, one example more.

Master. Here is another.

The first extracion of the diuisioz.

\[
48.3\overline{36}. - 20 \overline{c} - 24.\overline{c}. \quad (8. \overline{3}.3).
\]

3.\overline{9}. - \overline{6}.\overline{9}.

The remouynge for-ward of the diuisioz.

\[
48.3\overline{36}. - 20 \overline{c} - 24 \overline{c} (8. \overline{3}.3) - 4\overline{c}.
\]

5.\overline{9}. - \overline{6}.\overline{9}.

The comprobation of the same by resolution, acoumptyng still 2 for a roote.

\[
\overline{7}6\overline{8}. \quad 160 \quad 48.
\]

2\overline{8} - 6. (128.

The settynge for-ward of the diuisioz.

\[
\overline{7}6\overline{8} - 16\overline{8} - 48. \quad (128 \quad 12.8.
\]

Scholar. Yet ones again, I piae you woooke the like.

For although I perswade my self, that I perceive the woooke:yet would I see more confirmation of it, before I would be to constante in my persuation.

Master. Good advise. Semeste is ever sure: but if you doubte, your counselloure is not farre absente.

Scholar. I maie iustly rejoice thereof: But for ever mater to require ated, and never to travell my owne witte, it might seme mere daftardlinke. And so
The Arte

to were it plaine babishenelle, to count every mozel, to be chawed before hande, and put into my mouth.

Master. Then take this other example, in one platte complete: But with a cauet, to beware of to moche confidence, while you sene to sse doublefalle dasterblinckle.

\[ \frac{16}{2 \times 32} \]

Scholar. Now haue I, that I looked fo.

Master. Sothe, lette vs trie this woorkke, as wee haue done the other before we goo from it.

Scholar. I praye you let me doe it.

Master. With a good will.

\[
\begin{array}{c|c|c}
64 & 16 & 30 \\
14 & 30 & 480 \\
256 & 480 & 896 \\
64 & 16 & 32 \\
896 & \\
\end{array}
\]

Scholar. I kepe all the old roots

2. Then is the \( \frac{3 \times 32}{2} \), which being multiplied by \( \frac{14}{3} \), maketh \( 896 \).

And so, \( \frac{3 \times 32}{2} \), doe yelde \( 480 \). And

16. squares make \( 64 \). All thei togeth other yelde \( 1440 \).

The rest of the numbers, must be abated, because of the signes, and thei make \( 896 \)and \( 480 \).

\[
\begin{array}{c|c|c|c}
32 & 8 & 240 \quad \text{For every } \frac{2}{3} \text{ is } 32 \quad \text{and} \\
6 & 6 & 192 \quad \text{then } \frac{6}{3} \text{ times that, that maketh} \\
192 & 48 & 1440 \quad \text{whereunto I put } 48 \text{ fo} \\
6 \quad 6 & 120 \quad \text{Cubes: and so haue } 3.240. \text{to be abated out of } 1440. \text{and then remaineth } 1200. \text{fo} \\
\end{array}
\]

are \( 16 \) and \( 2 \) roots make \( 4 \),

\[
\begin{array}{c|c|c|c}
1200 & 120 \quad \text{If I divide now } 1200 \text{ by } 20. \\
20 & 60 & 1440 & 240 \\
\end{array}
\]

the quotiente will be \( 60 \). agreably \( 1200 \)
to the former quotiente. \( F0 \) \( 7.2 \) make \( 56 \)
And
of Cosike nombers.

And 8. rootes ye dome. 16. That is, 72. From which I
must abate. 3. 5. That is, 12. And then it is inuile. 60.

Mester. This is well done.

Scholar. Pea sure, I am perfect inough, in this
state of diuision, I trowe.

Mester. You doe well to doubt.

Scholar. I thinke my self sure without doubte: 
As by one or twoo examples, I will declare.

And first I take this nother 322 Z 5 2
— 42 Z 69 Z 30 Z .to be divided
by 14 Z 5 Z . Wherefore I sette them doune
thus.

\[
\begin{array}{c|c|c|c|c}
322 & 115 & 32 & 42 & 69 \\
14 & 5 & 14 & 5 & 30 \\
32 & 115 & 42 & 15 & 69
\end{array}
\]

And finde the firste quotiente to bee. 23, 3, by
whiche I multiplie the diuisor, and it taketh awaie
all the nombers over it: Wherefore I sette the diuisor
foward, & finde 3 Z , for the quotiente, whiche I mul-
tiplie into the diuisor, & it maiketh 42 Z — 15 Z .
Wherby I am at a stale. For although I see in the di-
vidende, the like nombers, yet the signe of —— de-
clareth, that it is not possible, to abate this newe no-
ber then: saynig —— 42 Z . is lesse then naughte.

Mester. Wherefore consider it, in cholyng your
quotiente: and giue your quotiente the like signe.

Scholar. But then riseth an other doubte. For
there will be —— 15 Z , whiche disagreeeth in signe
from the nomber over it.

Mester. Yet sate you subtracte it well thoughst,
if you have not forgoten, your rules of suctraction.

Scholar. Now I dooe better remember myself:
that by good reason, I must leave as a remainder, not
onyly the whole nomber over it, which is. 69, 5. 

but
The Arte

but I must addde therto 15. z. more.

So shall I cancell the. 69. and set over if. 84. And then do I remoue the diuisioz forwaerde, settyng 14 z. under. 84. z. and the rest in order, whereby I perceiue, that the newe quotiente will be. — 6. z.

\[
\begin{align*}
84. & \quad 84. \\
14 & \quad 14 \\
69 & \quad 14 \\
\hline
62 & \quad 15 \\
25 & \\
\end{align*}
\]

whiche quotiente I doe multiplie into the diuisioz, and it doeth make. 84. z. — 30 z. agreeable to the somme over it. And so there remaineth nothynge.

Master. You have done well. But in chysynge your divindende, and the diuisioz, your luckye was better thyn your cunning.

Scholar. That shal I pronue agayne, by an other example, takyn also at all adventures.

I would divide this somme.

\[
\begin{align*}
16 & \quad 16. z. \\
89 & \quad 20. z. \\
4 & \quad 12. z. \\
\hline
4 & \quad 89. \\
\end{align*}
\]

And firste I see, that 4. is contained in. 16. sower tymes: and so maie I finde. 2. in any other numbers there. 4. tymes. Wherfore I set. 4. in the quotiente.

And because the 4. in the diuisioz are. z. and the 16 to bee divided, are. z. C. accorcyng to the former rulles, I finde the newe denomination costike to be. 4. z. whiche
of Coских номеров.

Whiche I set in the quotient with 4 and so is it. 4. Z. Z.
Then saie I. 4. Z. Z. multiplied by. 4. Z. do make 16. Z. Z. and therefore cleareth and confirmeth al that come over it. Then farther saie I. 4. Z. Z. multiplied by. 2. Z. doe yelde 8. Z. Z. But I see noe soche deno- 

mination in the dividende.

Master. Then maie you perceiue, that you have 

missed.

Scholar. Why sir, I thinke I ought to doe as you 

did: that is to multiplie the quotient into every parte of the dividio.

Master. That is true: but I wil detecte the saute 

unto you. And that is this.
That all nombers Coских compunde, can not bee 

divided orderly, by dividioes compounde. And those 

that can bee divided, will not receive any other diul-

so of the same kinde, but one of. 2. nombers, by mul-

tiplication of whiche it was made: and so the other of 

those. 2. shall be the quotiente: As it came to passe in all 

those. 3. examples, which I set fo. the. And therefore it 

is loste labour, to goe aboute to divide theim in that 

softe.

Scholar. Then are there but fewe nombers of 

Coских compunde, that maie be divided.

Master. So many men saie. But I saie thereto, 

that though many of them can not be divided, by like 

nombers Coских compunde, yet are there many thou-

sandes, that maie be so divided.

And again I saie, that all sorte of theim, maie bee 

divided, by an Abstraft nomber. And also any of them 

maie be divided, by conversion into a fraction: And so 

maie your example be set thus.

\[
\begin{array}{c|c|c|c|c|c}
16 & .3 & .Z & .+ & 20 & .8 \hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
12 & .Z & .+ & 8 & .9 \hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
4 & .Z & .+ & 2 & .Z \hline
Z & j & \text{And}
\end{array}
\]
The Arte.

And in all other cases like, let the dividende over a line, and the divisor under the same line, and so is your division candied: and this is the reducible waie, and the moste indifferency, in all suche numbers.

Scholar. That is done learn'd. And therfore needeth no more examples.

It is like in numbers Abstracte, when the greater number, doth divide the lesser. As, 6. divided by 1 1. maketh 6.

Master. Somewhat like it is. Nowbe it here is a woone moare like thereunto, as when we should divide the lesser Cosike number, by the greater, for then we must set them in that soyme. So, 6. 2 divided by 7.. shall be set thus: \( \frac{6}{7} \). And, 2 0. divideth by 5. 2 \( \times \). must stande in this maner: \( \frac{20}{5} \).

Scholar. Why? 2 0. maie be divided by 5.

Master. But, \( \frac{20}{5} \). can not be divided by \( \frac{2}{5} \). And in Cosike numbers, the chief regard is to be had, to the Cosike signes.

Scholar. Then, as so any other soyme, of regular division, here is none.

Master. Hoe, excepte your divisor, bee a number Abstracte: For at the leaste, if it have one only Cosike signe, and be uncompounde, that signe must be other equalle, or lesser then the leaste Cosike signe, in the dividende.

For so, 6 0. 2 = \( \frac{60}{2} \). - - - - 4 8 = \( \frac{48}{2} \). - - - - 1 8 = \( \frac{18}{2} \). maie bee divided by any number, haung one of these 3 signes Cosike, 2, 2, 9.

Scholar. I understand it well. For, 2, is the laste signe in the dividende: And, 2, and, 9, are not only less then it, but also 9, leaveth the number, as if it were a number Abstracte.

So if I would divide your number, assigned by 4 0. 2, the quotiente would bee thus.

\( \frac{60}{2} \).
of Cosike nombers.

60.8 48.8 18.8 (1.8 8) 1.8 .8.
40.8 40.8 40.8

Master. Before we cane this worke of division, I will admonish you, of one case also, in the divisiou of diverse nombers. And that is, to consider, whether your dividende, doe omit any Cosike denominations, betwene them, which it hath. For, if it doe, you must yet supplie their rooms, with signes and Ciphers. As by example, you shall understand.

I require to have this number. 8.8. + 64.8.
divided by. 2.8. + .4.8.

Scholar. That will I doe quickly. For I see. 4. 
will be the first quotient, and his denomination will be. 8.8. 8th. 8. divided by. 2.8. doe make 8.

But first I sette downe the numbers orderely. And then I multiply the divid.
For by the quotient, & there riseth. 8.8. + 16.8. 2.8. + .4.8.

Master. Stande you now amazed, for all your great confidence: You see that you can not finde any 8. in the dividende. Wherefore set downe the number as I told you before, in this sohte.

--- 16.8.
8.8. + .8. + .8. + 46.8. (4.8.
2.8. + .4.8.
--- 8.8. + 16.8.

And then I take the same quotient that you did, and finde the remainder to be. --- 16.8. Wherefore I doe again sette forward the divisor: And finde the quotient to bee. --- 8.8. by whiche I multiple the Z.8. divisor,
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divisio; and it maketh. 16. \( \frac{32}{2} \) to that abatnyng the. 16. \( \frac{32}{2} \) the reste, that is, \( \frac{32}{2} \). Shall be the remainder with the signe, + by the rule of subtraction.

\[
\begin{align*}
16 & \div 32 \\
5 \cdot 32 & \div 64 \div 8 \div 16 \\
2 & \div 4.
\end{align*}
\]

Then under that remainder, I remove the divisio; and finde the newe quotiente to bee, = 16. 9. And so is the number cleerely consumd.

Scholar. If I forgette any parte of this, I am deceiued to soule.

Saker. Then haue you learned this parte, well enough, for this tyme. And therefore will we goe thowth unto fractions, which partly were omitted before, and partly are compounde of them self.

Of fractions, and their numeration.

Fractions of this kinde appeare simple; and yet are counte so to bee judged: as \( \frac{\frac{\frac{4}{2}}{2}}{2} \) betokeneth 4. \( \frac{2}{2} \) to bee divided by. 3. \( \frac{2}{2} \). Likewise this fraction \( \frac{\frac{12}{3}}{3} \) doeth import that 12 \( \frac{3}{3} \) must be divided by. 5. \( \frac{5}{5} \). But \( \frac{3}{3} \) betokeneth. 10. \( \frac{10}{10} \) to bee parted into. 19. portions.

And here shall you note, the doubtfull soyme, that many menne in this arte use, whiche write that laste fraction thus, \( \frac{19}{10} \). Where as this fraction doeth represent \( \frac{19}{10} \) of a square: and not 10. \( \frac{10}{10} \) to be divided by. 19.

Scholar. Because you saie, that some doe so use it, and
of Coßike nombers.

and I would gladly excuse all good writers: I maie saie for them that as in bulgare nombers, when 1 0. should be diuided by 1 9. And is set thus: it dooth insipe bothe that 1 0. is diuided into 1 9. and also that every portion of those 1 9. is 1/3 of an unitie: so that if 1 0. e. should be parted emongest 1 9. men, every man should have 1/3 of 1 e.

Master. Your wordes have so moche apperauce that they maie persuade hym, that is not very precise in termes, especially syping there is no other quotiente there, but the same nomber. But as the somme of 1 0. e. bynng diuided by 1 9. is farre more then 1/3 of an unitie: so 1 0. 3., to bee diuided by 1 9. differ moche from 1/3 of a square. For the one is 1 9. tymes so moche as the other. And therfore oughte to have a distincte somne in wrytyn.

Scholar. Then you would have me to write the so, that 1/3 of a Square, should have the signe against the line, as here is set 1/3: and when I would reresent 1 0. 3., diuided by 1 9. I shall write it thus: 1/3 with the signe above the line.

Master. You maie see their agremente, and their difference by resolution, in this maner 1/3 will make 1/9 accouemptynge. 2. fo a roote, and 1/3 3., maketh 1/9 of 4.0. 4. of 1.

Again, accouemptynge 3. fo the roote, then 1/3 vel-deth 1/9: and 1/3 maketh 1/9 of an unitie: so the appere to bee equall in valewe by reduction.

But now maie you see, that the one dooth betoken the firste nober, whiche is to be diuided: and the other dooth signifie the quotiente of the division: and so are thei distinate in office and nature. But because by resolutio, the one tourneth into the other, therefore manie men accouempt them as one. Howbeit, we stand to longe abouthe this, consideryr the erroure, is not alwaies daungerous.
But their oversight is more dangerous, which misplace the signe, when it should be sette under the line: as a great clerk doeth (except I shall so; his excuse, impute the fault to the printer) so he meaning to divide 3 by 7. \( \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \), where he should write it thus \( \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \). and againe, myndynge to divide 7 by \( \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \), where he should write \( \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \).

Scholar. This fault is manifest, and detecteth the first negligence: For \( \frac{1}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \), doeth make in number, after the former resolution. \( \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \) doeth make \( \frac{1}{7} \).

Master. Well, seyng you perceive the faulte, we will stande no longer aboute it. Therefo to proceed distinctly and certainly, whether that fraction be compound, or simple, where the numeratour is a Cofike number, and the denominator, a number absolute, yet make you boldly thinke, that fraction to bee compound, whose numeratour is a number Cofike and the denominator an other Cofike of unlike signe: as \( \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \).

Yet as in numbers Abstracte, it make some more aptly to bee called a fraction, when the numeratour, is lesser, then the denominator, so in numbers Cofike, more aptly the signe of the denominator, should bee the greater. Yet both the formes come in use.

And so, because casiness in woorkyng, doeth oftentimes bringe certaintie with it before we take in hade the addition of fractions, I thinke it good to speake somewhat of Reduction, to another denomination. So that you forgette not, that any 2 numbers Cofike compounde, with a line betweene them, maie be called a fraction. As thus \( \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \); that is, \( 5 \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \) to bee divided by \( \frac{1}{7} \).
Examples of Numeration.

3.5. + 1.2.9. and so of other like.

Of Reduction of fractions.

Fractions Cossike, not only in their
numbers, but also in their signs
may be reduced to other valuations,
and namely to their leaste terms,
and yet continue still in one propor-
tion, betweene the numerator, and
the denominator.

So 2/5 may bee reduced to 6/15: so to high as c
is above. g, that is in the third place from it: so is
g^3/5 in the third place above. g.

Againne, 3/2 by reduction doth make 3/6: And so
will bee by reduction.

And so in all other fractions, where the numbers bee
commensurable.

But if any one number, bee incomensurable with
the other, then can there be made no reduction in the
numbers. Yet in the signes Cossike, there may be a re-
duction, other to greater, or to smaller signes: For
those signes be ever commensurable.

And there is no exception, but their may bee redu-
ced to smaller quantities, excepte any one quantitie
of thoem bee. g, that is a number. For that can bee no
smaller. And therefore none other may bee altered, but
every one must be abated alike.

And looke how moche, the smalleste quantitie of
that fraction, is above a number, so moche may the
all bee abated: so they are never reduced to the small-
lest, till one of them be a number.

Scholar. And why may not this reduction, serve
for whole Cossike numbers?

Master. Because the whole number, doth not co
The Arte

first of a proportion, as the fraction doeth, and so male bee expressed in diverse terms: but it importeth one comme certaine, whiche male neither bee increased, no: decreased, but it will change his value, and alter his office.

And if I saie: a foote is \( \frac{1}{3} \) of a yarde, I male saie as truely, increasing both the numbers, in the like proportion, a foote is \( \frac{1}{4} \) of a yarde: or in lesser terms: a foote is \( \frac{1}{3} \) of a yarde.

But when I saie in whole number, a yarde is 3. foote, 2. foote is 12. inches, I saie truely: and if I doe increase or abate any of those numbers, my words will be false.

So although in this number, 8.\( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). by reason of both numbers and signes, there might bee a reduction, yet because it is a whole number, it should thereby bee abated moche; as here you male see. 4.\( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). whiche by resolution into vulgar numbers, 2. being sette as the root, doeth make. 32. \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). that is. 3.3. and the other number before, doeth yelde by the like resolution. 25 6. \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \). that is 2.64. and is 8. tymes so moche as the other.

Scholar. I perceiue now good reason, why reduction srueth for fractions onely. And if there bee noe more difficoltie in it, then you have declared. I can worke it easie.

Reduction in For other the reduction consisteth in the signes Cos. signes onely, as \( \frac{1}{2} \) \( \frac{1}{2} \) \( \frac{1}{2} \) where the numbers bee uncommensurable, and therefore can not bee altered to any lesser terms. But the signes Cosike maie bee abated by 3. denominations: semyng the smalleste of them, is so many in order above. \( \frac{1}{2} \). And therefore it maie be reduced to \( \frac{1}{2} \).

Reduction in Other els secondarily, the reduction consisteth in numbers onely the numbers onely, when the numbers be communi-
of Co\$\$ike nombers.

eante. And the signes Co\$\$ike bee all reddy at the leaste:
as when one of them is $\frac{5}{2}$. So $\frac{5}{2}$ will bee reduced
to $\frac{5}{2}$.

Czels thirdly, the reduction maie bee wroughte, Reduction in
bothe in signes, and also in nombers. When all the
signes be aboue $\frac{5}{2}$ and the nombers be communicant
So $\frac{5}{2}$ may bee reduced well unto $\frac{5}{2}$.

Mastur. Yet one so,me of reduction mo\$re, I will An other
showe you, where not onely the like woorke maie bee,
but also the nomber maie be broughte from his com-
position, to a mo\$re simplicitie, by abatyng some of
his partes.

As this nomber $\frac{5}{2}$ may bee reduced,
firste by his nombers to $\frac{5}{2}$.

Secondarily, by his signes it may bee altered thus.

Thirdely, by abatyng the nombers, that lowe the
signe of compositie (that is $\frac{5}{2}$) it may bee brought
to $\frac{5}{2}$. Which fractions, kepe the self same
proportion, that the firste fraction did.

Like waies with the signe of --- nombers resi-
dualles, may bee reduced. As $\frac{5}{2}$ will bee
reduced, as the other was to $\frac{5}{2}$.

Scholar. This is unto me a maruelouse mater,
that those 2. contrary nombers, should be reduced to
one fraction.

Mastur. The like happeneth in vulgar nom-
bers. For $\frac{5}{2}$ will bee reduced to $\frac{5}{2}$. For firste
it make $\frac{5}{2}$ and then $\frac{5}{2}$. So like waies $\frac{5}{2}$
will make firste $\frac{5}{2}$ and then $\frac{5}{2}$.

And the reason of it, dooth depende of the 19. pro-
position, of the fift booke of Euclide, where it is writ-
ten thus.

Aaj. If
The Arte

If the proportion of the abatemente vnto abatemente be, as the whole is in proportion to the whole. Then shall the residue bee in like proportion to the residue, as the whole is to the whole.

That is in the last example. As. 18. is vnto. 24. so is 6 vnto. 8. Therfore shall 12 be to 16, as 18. is to 24.

And so to exercise you the better, loe, here are one or two examples more, of the like reduction.

\[ \frac{7c}{8} = \frac{14c}{16} \]

maketh \[ \frac{7c}{8} \] of \[ \frac{7c}{8} \]. Again \[ \frac{12c}{16} = \frac{42c}{52} \].

Yelbeth \[ \frac{12c}{16} \] of \[ \frac{42c}{52} \].

But this woulde you farther marke, that in Cosiike numbers, not onely the numbers, but also the Cosiike signes must bee, according to Euclides proposition.

Scholar. That doe I see.

Foe in the last example: As. 9. is to. 3, so. 3c. is to. 3.

And in the next example before: As. 3c. is to. 3c.

so is. 3. to. 3.

Likewaies in the other examples, as \[ c \] is to \[ c \] so is. 3. to. 3.

At this is good and reasonable.

Master. How doe you see, bothe the maner of re-
duction, and also some reason foe it. Therfore I will
procede, to declare the wooste of Addition.

Of Addition and Subtraction.

In Addition there is nothynge moare, then you have learned before: Foe as foe the multiplications of the denominatoz together, and then crosse waies with the numeratorz of theother, is yelte agreeable with the reductions of Ab-
tracte fractions, to byng them to one common de-
nominator.
of Coöike nombers.

And then the numerators added together, dose make the newe numerato in addition.
And likewaies the lesser numerato, subtracted fro the other, doeth make the numerato in subtraction: wherefor a fewe examples maie suffice.

Examples of Addition.

\[
\begin{array}{c}
\frac{4}{3} \text{ to } \frac{4}{3} \\
48
\end{array}
\]

That is in smaller terms.

\[
\begin{array}{c}
20 \text{ to } 21 \\
24
\end{array}
\]

Here you see how the fractions be sette betwene 2 lines: and under the nethermoste line, is sette the newe denominator: and over the higher line, are set the newe numerators joyned in one.
The firste of them, can not be reduced to any smaller terms, because the numbers be not all 3 commensurable: s the denominator, also is a number Abstrait.
The seconde hath also a number Abstrait for his denominator, and therefore there can be noe reduction in signes; but the numbers all 3, being commensurable, s divisible by 2, maie be reduced, as there you see.

More examples of Addition.

\[
\begin{array}{c}
16 \text{ to } 4 \\
20 \text{ to } 5
\end{array}
\]

Any That
The Arte

That is in smaller termes.

\[
\frac{4.9}{5.6}
\]

Here is noe multiplication wrought, because the denominators are like.

Another Example of Addition.

\[
\begin{array}{c}
5.8 + 2.0 = 3.8 \\
5.8 + 3.6 = 9.4 \\
\hline
6.4 + 2.0 = 8.4 \\
\hline
6.4 + 3.8 = 10.2
\end{array}
\]

That is in smaller termes.

\[
\frac{5.8}{6.4}
\]

Here is noe multiplication, no reduction to one common denominator: with that be one all ready: neither can the numbers be reduced to any other letter: but the quantities only be reduced as you see.

Scholar. I praise you let me prove.

Another Example.

\[
\begin{array}{c}
8.0 + 9.0 + 6.0 = 30 \\
8 + 9.3 = 17.3 \\
\hline
10 + 3.8 = 13.8 \\
\hline
110.8
\end{array}
\]

That is

Mater. Marke your worke well, before you reduce it.

Scholar. I see my faulte: I have sette 2. numbers severally, with one signe Cosike: by reason I did not foresee, that \( \times \) multiplied with \( \times \) doth make the like
of Cosike nombers.

Like quantitie, as $\frac{1}{2} \times \frac{1}{2}$, multiplied by $\frac{1}{2}$. Therefore it should be thus.

\[
\begin{align*}
80. \times \frac{1}{2} & = 1.50 \times \frac{1}{2} - 30. \times \frac{1}{2} \\
& = 110. \times \frac{1}{2}
\end{align*}
\]

Whiche maie bee reduced, by meane of the nombers, to this comma.

\[
\begin{align*}
8. \times \frac{1}{2} & = 1.50 \times \frac{1}{2} - 3. \times \frac{1}{2} \\
& = 11. \times \frac{1}{2}
\end{align*}
\]

And now considerying the Cosike signes, and worკyng as I have marked you to dooe: That is to abate the leaste signe, out of theim all: because $\frac{1}{2}$, is here the leaste, I abate it out of $\times \frac{1}{2}$, and there resteth $\frac{1}{2}$, and so doing with the other signe, $\frac{1}{2}$, there remaineth $\frac{1}{2}$ then $\frac{1}{2}$ out of $\times \frac{1}{2}$ doeth leave $\frac{1}{2}$, and none: So will the fraction bee thus: $\frac{8}{11} \times \frac{1}{2}$ by reduction in signes and nombers also.

Wherefore. Sayng you have so well marked the reduction of the signes (whiche followeth the forme, taught before in division) I thinke it not needfull, to name any longer aboute this.

Wherefore we will goe forwad to subtraction, after that I have admonished you of fractions, in appearance simple, whiche by doce by addition, bee come compundde. As this $\frac{1}{2}$, added to $\frac{1}{2}$, maie firste be added by the common signe of addition, thus.

\[
\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{2}
\]

Whiche by reduction, unto one denomination, will be thus written, $\frac{1}{2} - \frac{1}{2}$.

But as this is cawse enough to understand, so maie it helpe often times, for specie worke, as well in additiō, as in subtractiō, by the onely adying of the signe.

As if I would subtrahcte this fraction $\frac{1}{2}$, out of Subtraction.

\[
\begin{align*}
& = \frac{1}{2} \times \frac{1}{2}
\end{align*}
\]
The Arte

\[ \frac{2}{5} \times \frac{2}{5} = \frac{4}{25} \]
And so is the Subtraction woughte.

Yet make you reduce them, to one denomination, if you will, after the same forme, as you did in addition. And then will it bee, \( \frac{3}{5} - \frac{2}{5} \) which can not bee reduced to any smaller termes, because the numbers are not commensurable: and one of them (that is to saie, the denominator) is a number Absurd.

Scholar. I see in this, there is no difference from Addition, but in the signes, and wherfore I will proue an other example, by your leave.

I would subtracte \( \frac{1}{2} \frac{1}{2} \) out of \( \frac{4}{4} \frac{1}{2} \), and it will bee at firste \( \frac{1}{4} \frac{1}{2} \), and by reduction \( \frac{1}{4} \frac{1}{2} \).

Master. Your woorke is well done, accordingly to your firste meaning: But as the numerator of this laste reduction doth declare, it can not bee well, that \( \frac{1}{3} \frac{1}{2} \) maie bee abated out of \( \frac{1}{2} \frac{1}{2} \). For the greater absolutely, can not well be abated out of the lesser: and therefore you might rather have abated \( \frac{1}{2} \frac{1}{2} \) out of \( \frac{1}{2} \frac{1}{2} \).

Scholar. I see it well now: for the \( \frac{1}{2} \) is always double \( \frac{1}{2} \) triple, \( \frac{1}{2} \) yet more times greater, then the \( \frac{1}{2} \). Because the \( \frac{1}{2} \) commeth by multiplication of the \( \frac{1}{2} \) by his firste roote.

Master. Yet here in is discretion to be vsed, for in fractions, sometyme the number of the greater signe maie be the lesser. As for example, \( \frac{1}{2} \frac{1}{2} \) is less then \( \frac{1}{2} \frac{1}{2} \), as by resolution you maie proue, accounting 2 for the common roote.

Scholar. 2. being the roote, \( \frac{1}{2} \) is the \( \frac{1}{2} \), and his \( \frac{1}{2} \) maketh \( \frac{1}{2} \), then \( \frac{1}{2} \frac{1}{2} \), being \( \frac{1}{2} \), dooth appere double to it: and therefore greater by moche.

If I doe by the like resolutiō, proue the other fractions before, \( \frac{1}{2} \frac{1}{2} \), will bee. 2 \( \frac{1}{4} \) and \( \frac{1}{4} \frac{1}{2} \) will bee \( \frac{1}{2} \frac{1}{2} \): which is less then moche.
of Cosïke nombers.

So I perceive the greatnese and smallnese of the fractions, must be considered, as well in the numbers as in the Cosïke signes. And farther, if their fractions be nigh of one greatnese, or the fraction of the lesser signe the greater, then can not the subtraction, appeare reasonable.

Master. That is true, if those 2 fractions stande alone: els beynge partes of other numbers, it make appeare reasonable enough. As in this example of compound fractions. \( \frac{1}{5} \frac{2}{3} \) maie bee abated out \( \frac{2}{3} \) \( \frac{3}{5} \), and yet in the abatement after not onely the number \( \frac{2}{3} \) is greater, than \( \frac{3}{5} \) in the other, but also, the Cosïke signe \( \frac{3}{5} \) is greater then the other Cosïke signe \( \frac{2}{3} \).

Scholar. I consider it to be so: and yet \( \frac{3}{5} \) doeth so moche exceed \( \frac{2}{3} \), that it supplieth sufficiently the other defaulte: els could it not be well done.

But for this wooske, I must crave your helpe: because I have not seen the like.

Master. You maie doe in this, as I said before, generally for all subtractions.

Set doune bothe the numbers in due order, so that the abatement dooe solowe in order: and putte betweene them the signe of subtraction: as thus:

\[ \frac{1}{5} \frac{2}{3} - \frac{3}{5} = \frac{1}{5} \frac{3}{5} \]

Howbeit, if you will firste reduce every compound fraction, into one fraction, it will seeme more apt.
As thus. \( \frac{1}{5} \frac{2}{3} \) beynge reduced by additio will make \( \frac{11}{15} \), and by farther reduction of numbers. \( \frac{11}{15} - \frac{13}{15} \). Likeways \( \frac{2}{3} \) \( \frac{3}{5} \) will make by the firste addition. \( \frac{11}{15} + \frac{13}{15} \) and by farther reduction \( \frac{11}{15} \).

Now toyme them together, with the signe of subtraction, and they will stande thus:

\[ \frac{1}{5} \frac{2}{3} - \frac{3}{5} = \frac{11}{15} \frac{13}{15} - \frac{11}{15} \frac{13}{15} \]

Scholar.
The Arte

Scholar. This dooth appeare verie straunge unto me; but by vs I shall finde it more familiar: Seeing I see the reason of this worke, to agree with the worke of common fractions.

But so the proof of it, I will resolue eche worke, into numbers absolute, accounting 2 for a roote.

Mastar. So, shall you finde it true: But so called worke, take rather, 10, for the roote.

Scholar. I thank you, for your aide.

Then if 10 be the roote, the square will be, 100. and the cube, 1000. Now \( \frac{1}{2} \) of 1000, is 600. And \( \frac{1}{4} \) of 10, which is the roote, will bee 6. Whiche bothe put together, doe make 606. and that is the greater number.

Then so, the letter \( \frac{1}{16} \) of \( \frac{1}{2} \) of 1000, arc in this example. 400. For the cube being 1000, his \( \frac{1}{16} \) is 100. Again the square being 100, \( \frac{1}{4} \) of 100, must needs bee 75. Whiche being put unto 400, doeth make 475.

Then doe I abate, 475, out of 606, and there will reste, 131. How now.

Mastar. I perceiue you sate, as being astonisheth, because in the former worke, there is not leste a remainder: But the 2. firste summes enely altered by reduction, and joined together, with the signe of subtraction: Wherein if you had continued your worke, you should have founde the same numbers.

For 3, must needs bee 3000. For the 1, is a 1000. And also, 32, are 30: Whiche bothe added together, make 3030. Divide them by 5. (as the denominator would) and it will bee 606, as the valewe of the firste fraction.

Then come to the later number: and you may lene thinke that 8, are 8000. And 15. Squares are 1500. adde them together, and they will make 9500. Whiche must bee divided by 20. (as the denominator)
of Cozike nombers.

minatoz (importeth) and there will amounte. 475. the balewe of the lesser fraction: whiche nombers appeare the same, that were before: and thereby the wooke is good.

But if you will byng it to a remainder, doe thus. Reduce these 2. newe fractions, into one denomination.

Scholar. That can I doe, by multipliyng the numeratoz together: that is, 2 o. by 5. and thereof cometh 1 o. whiche shall be the common numeratoz: then must I multiple in crosse wares, the numeratoz of the firste, by the denominator of the seconde, and contrarily.

So for the firste numeratoz 3. &. 2 o. I woo:ke thus. And thereby dooth amounte (as you see) 6 o. &. 6 o. &.

6 o. &. 6 o. &. And so for the seconde numeratoz, I multiple, 8. &. 1 5. by 5. and there dooth rite. 4 o. &. 7 5. &. eche of theym hauing one common numeratoz 1 0 o.

Wherefore, leyng bothe nombers, haue one denominaeto, I shall abate the lesser numeratoz: out of the greater, as here in example is set fo:the: and then the

\[
\begin{align*}
6 o. &. 6 o. &. \\
4 o. &. 7 5. &. \\
\hline
2 o. &. 6 o. &. 7 5. &. \\
\end{align*}
\]

remainder will bee (as you see). 2 o. &. 6 o. &. 7 5. &. unto whiche I muste adde the common denominator 1 0 o. and it will be thus.

\[
\begin{align*}
2 o. &. 6 o. &. 7 5. &. \\
\hline
10 o.
\end{align*}
\]
Now prove whether this remainder, doe not agree to
thother remainder before, in your trial: Which was 13

Scholar. 2000 do make, 2000.600. yeeld 600: those 2 sommes I muste addde together, because of
the signe. — and it will be. 20600, then. 75. $.
are.7500. which I must abate from the
former somme of, 20600, and there will
remaine. 13100. for the numerato2, and
100. for the denominato2, thus. $1100$. $13100$

Maste. And what doe you thinke of it?

Scholar. By that I learned in the vulgar fractions, I knowe that it is inuste, 131, and so dooth it a-
gree precisely, with the former proofoe.

Maste. Well yet for more exactnesse in this
work, I will farther reduce that fraction, by divid-
ing the numerato2 by the denominato2: wherfore 200.
divided by. 100, doeth yeeld, $\frac{1}{2}C$. And 60. $\frac{1}{2}C$, di-
vided by. 100, doeth make $\frac{1}{3}C$. And lastly, 75. $\frac{1}{2}C$, di-
vided by. 100, will yeeld $\frac{1}{4}C$. So is the same fraction so reduced $\frac{1}{7}C$. $\frac{1}{2}C$. $\frac{1}{4}C$. And now doe
what that is, by the former proofoe.

Scholar. I make sone perceve, that $\frac{1}{7}C$. is, 200.
when the Cube is. 1000: And so $\frac{1}{3}C$. is, 6, whiche I
must addde together, and it will be. 206. Then $\frac{1}{2}C$, is
75, whiche if I doo abate from. 206, there will re-
maine. 131, agreeably as before. And so is this work
fully examined.

Maste. Yet will I propounde one or two ex-
amples more, partly to practize your memoe, and part-
ly to admonishe you, if you happen to see any soch
misfrook, in some other bokes (as I have doen) how you maie amende the erroore, and not stale at it.

Firsfe take this example. I would subtrac
t.

\[
\begin{array}{c}
\text{489.} \\
123 \\
\text{35.} \\
\end{array}
\]

out of \[
\begin{array}{c}
\text{489.} \\
\text{75.} \\
\end{array}
\]

Scholar.
of Cosike nombers.

Scholar. I must first multiplie the denominatoze together, and so it will make, as here is sette sooz the
84. č. — 21. z. z.

Then I multiplie the numeratoze of the firke, by the
denominatoze of the seconde,
and it will byng
48. z.

sooz the 336. z. which is the numeratoze
7. z.
for the abatemente.

336. z.

Afterward I multiplie the numeratoze
of the seconde,
by the denominatoze of the
firke, and it will make
576. č. — 144. z.

Now if I subtrate that
336. z. 
— 144. z. it will bee
576. č. — 480. z. for the abatemente that should
be subtrated now, is sette after the signe — with
the former somme of 144.

Finally, to make the remainder complete, as that
laste number is the numeratoze, so unto it I must adde
the common denominatoze 84. č. — 21. z. z.,
and it will bee 576. č. — 144. z.

that is in letter termes

Master. Now prove your cunning in this somne,
— 160 č.
— 21 č.
— 84 č.

Scholar. Firste I must reduce them, to one com-
mon denominatoze, by multipliyng bothe denomina-
84. z. — 21. č.

1008 č. — 252. z. z.
63. č. — 252. z. z.
— 63. č.
— 1008 č. — 504. z. z.
The Arte

toys together. And so wilt it be. 63s 3x — 1008 £
— 504 £. as by speciall wooske, I have here
proved.

Then doe I multiplye the numeratoz of the totall,
by the denominatez of the abatemente, as here also I
have particulrly set fo the in wooske, for my owne
ease, and avoiding of erroure: And so I finde it to be
1056 £ — 6912 ze — 696 £. which
shall be the numeratoz of the totalle.

\[
\begin{array}{c}
232 ze \quad \text{——} \quad 576 z. \\
12 ze \quad \text{——} \quad 3 z.
\end{array}
\]

\[
\begin{array}{c}
2784 ze \quad \text{——} \quad 6912 ze. \\
696 £ \quad \text{——} \quad 1728 £.
\end{array}
\]

\[
\begin{array}{c}
1056 £ \quad \text{——} \quad 6912 ze \quad \text{——} \quad 696 £.
\end{array}
\]

Then doe I multiplye the numeratoz of the abate-
mente, by the denominatez of the totalle which thing
is easely done, because the one nomber, is a nomber
Anstates) and so have I for the numeratoz of the ab-
temente. 4032 £ — 1008 £.

And semyng these two nombers, have one common
denominator, I shall abate the lesser numeratoz, out

\[
\begin{array}{c}
1056 £ \quad \text{——} \quad 6912 ze \quad \text{——} \quad 696 £. \\
4032 £ \quad \text{——} \quad 1008 £.
\end{array}
\]

\[
\begin{array}{c}
6912 ze \quad \text{——} \quad 312 £ \quad \text{——} \quad 2976 £.
\end{array}
\]

of the greater, so will there be left for the numeratoz
of the remainder 6912 ze — 312 £ — 2976 £
unto whiche, I shall adde the common denominator,
and then will it be.

\[
\begin{array}{c}
6912 ze \quad \text{——} \quad 312 £ \quad \text{——} \quad 2976 £. \\
63s 3z \quad \text{——} \quad 1008 £ \quad \text{——} \quad 504 £. £.
\end{array}
\]

That
so Cosike nombers.
That is in letter terms.

\[
\begin{array}{c}
2304.\text{z} - 104.\text{z} &= 992.\text{z} \\
21.\text{z} - 336.\text{z} &= 168.\text{z}
\end{array}
\]

Master. You haue wroght it well. And hereby I conjecture, that you are experte enough in subtraction. Wherefore now we will goe in hand, with multiplication and division.

Of Multiplication.

And firste, concerning multiplication, here is no more to bee saide, cation, then hath been taughte before.

For the numbers shall bee multipliied, as common fractions are wonte to bee: that is to saie, numeratoz, by numeratoz, and denominatoz, by denominatoz.

And for the chaunge of their denominations Cosike, the rules given before shall suffice: So that a fewe examples shall sufficiently instruct you, in the worke of it.

As this for the firste.

\[
\begin{array}{c}
20.\text{z} - 19.\text{z} \\
6.\text{z} &= 3.\text{z}
\end{array}
\]

\[
\begin{array}{c}
120.\text{z} - 114.\text{z} \\
60.\text{z} &= 37.\text{z}
\end{array}
\]

\[
\begin{array}{c}
120.\text{z} - 114.\text{z} - 60.\text{z} &= 57.\text{z}
\end{array}
\]

Where I dooe multiplie.

\[
\begin{array}{c}
20.\text{z} - 19.\text{z} \text{ by } 6.\text{z} &= 3.\text{z}
\end{array}
\]

\[
\begin{array}{c}
31.\text{z} - 4.\text{z}
\end{array}
\]

And soke I shall multiplie, numeratoz by numeratoz:
The Arte.

ratio: where 20 : 5, multiplied by 6 : 5, doth make 120 : 5 as the former table of multiplication, for chang: of Cos,ike signes doeth declare. And so in all the reste, there is no dificultie, if you remember that, that you have learned before.

Scholar. I perceiue it well. And so the whole newe numerato: will bee. 120 : 5, — 114 : 5, = 60 : 5, — 57 : 5. And the denominator will bee. 124 : 5.

So will the whole fraction bee.

\[
\frac{120 : 5}{124 : 5} = \frac{114 : 5}{60 : 5} = \frac{57 : 5}{124 : 5}.
\]

That is not to bee reduced to smaller termes of numbers, because thei be uncommensurable, but in Cos,ike signes, it mighte bee boughte to one letter, as.

\[
\frac{120 : 5}{124 : 5} = \frac{114 : 5}{60 : 5} = \frac{57 : 5}{124 : 5}.
\]

Now will I proove an other number, as fortunate doeth offer it to mynde. That is \(\frac{12 : 5}{21 : 5}\), to be multiplied by \(\frac{13 : 5}{36 : 5}\).

An Absurde Master. It appeareth: that you take theim, at all adventures. For: your firste number, semeth to be an preseth lesse Absurde number. Seuing his numerato: is lesse then naught, in appearance. And then maie it not bee dividde by any number: and moche lesse by so great a denominator.

Scholar. It is easie to see, now that I am admonished thereof. For: it is not possible, that any Surfolido number, can bee lesse then power times moche, as the Cube of the same nature. Seeping: every Surfolido is made, by multiplying the Cube by the square of the like Roote, but lesse then. 4. is there no Square. And therefo:re every Surfolido, doeth exceede his Cube power times at the leaste.

20
of Co$ike nomers.

So that, $\frac{1}{2} \cdot 3 = \frac{3}{2}$, were nothing, and so is an Absurde nobr. And therefore, $\frac{1}{2} \cdot \frac{3}{2}$, is moche lesse then nothing, and is therby an Absurde nomber also.

Master. Yet maie your example scrue, to teache and practise multiplication by, as well as any other.

And farthermore, I will tell you by this occasion, that I spake to you, more after the opinion of the co- mon number of artes men, then after my owne judg- ments.

Scholar. I might thinke so, by termynge of your sentence; but yet was your saying true.

Master. Yet maie that fraction stand well, if you take a broke nobr Abstraete for the roote. Although in whole nombers, it bee an Absurde nomber.

Scholar. That will I prove, by settyng $\frac{1}{4}$, for a roote. Then will the Square be $\frac{3}{4}$, and the Cube. $\frac{27}{64}$. Also the Square of squares will bee, $\frac{1}{8} \cdot \frac{1}{8}$, And the Sursole $\frac{343}{729}$.

And now to prove by resolution, how my number will rise, I take $\frac{1}{2} \cdot 3 = \frac{3}{2}$, that is $\frac{8}{13}$, or $\frac{13}{8}$, which I note as the firste somme. Then I take like waies $\frac{3}{2}$, which yeldeth $\frac{583}{217}$, that is $\frac{6}{13}$. And now I see that I maie abate it very well, out of $\frac{13}{2}$.

Master. So maie you see, that as in whole nombers, euere moare the greater Co$ike$ signes, will have the greateste nombers: So in fractions resolued by Co$ike$ signes, the greatest fraction, amounteth to the leaftest signe: and the leaftest fractio, agreeth to the greateste signe.

The reason of it is this. That the moare any fracti- tion is multiplied by a fractiion, the leater it wareth. For as whole nombers by multiplication, maie in- crease infinitely; so fractions by multiplication, maie decrease
decrease infinitely.

But before wee passe from multiplication, I will pour you with one example moare. I would have \( \frac{13c}{78} \) multiplied by \( \frac{13c}{58} \).

Scholar. I am troubled with the multiplier. For I knowe not what to make of it.

Master. You doubt (I thinke) of the numeratio of it, because you had not the like example before: for it is a mirte number of a fraction, and a whole number. But leyng the signe of abatemente is set against the whole fraction, and nother agayne the numerato, nor denominator, therefore must that 4. \( c \) be understoode, to be abated out of the full fraction.

Scholar. Now I perceiue the mater. For there might be 3. diverse soymes, to place that abatemente.

As here I have set them. \( \frac{14c}{35} \) \( \frac{24c}{35} \) \( \frac{14c}{58} \) \( \frac{14c}{35} \).

And as it was set by you, \( \frac{24c}{35} \) \( \frac{24c}{35} \). Which I wil resolue into absolute numbers, to see their difference the better. And so, taking 3. for the rootes, these will be their 3. soymes.

Foz the firste \( \frac{648}{81} \) \( 02 \) cls \( \frac{616}{81} \) that is \( \frac{112}{17} \).

Foz the seconde \( \frac{648}{81} \) \( 02 \) cls \( \frac{648}{81} \) that is \( \frac{116}{27} \).

And for the thirde nomber, which is our specialle nomber. \( \frac{648}{81} \) -- 12. that is 8. -- 12. and is an Absurde nomber. Foz it betokeneth lesse then naught by 4.

Master. If you would haue it no Absurde nomber, you must increase the proportion of the fraction, by augmentinge the numerato, or abatyng the denominator, or cls thirdly by abatyng the nomber, after the signe of abatemente. As \( \frac{24c}{35} \) \( 4. 8 \) : 02 cls secondarilie, thus. \( \frac{24c}{45} \) \( 4. 8 \) 02 thirdely \( \frac{24c}{45} \) \( 4. 8 \).

Howbeit for examples sake, you maye woorke, as well with Absurde nombers, as with any other.
of Cosike nombers.

But so you case, I will shewe you the woorkke of this example, in two foymes.

First, you shall multiply the firste whole nomber, by the fraction of the seconde nomber, that is. \( \frac{15}{8} + \frac{12}{3} \) by \( \frac{3}{2} \), and it will bee.

\[
\begin{array}{c}
456.3\overline{c}. \quad + \quad 72.3\overline{c}.
\hline
63.3\overline{c}.
\end{array}
\]

As here in woorkke you maye see it plaine.

\[
\begin{array}{c}
19.\overline{c}.
\hline
24.\overline{c}.
\end{array}
\]

\[
\begin{array}{c}
456.3\overline{c}.
\hline
72.3\overline{c}.
\end{array}
\]

That is in lette termes, bothe of nombers, and of signes Cosike.

\[
\begin{array}{c}
152.3\overline{c}.
\hline
24.\overline{c}.
\end{array}
\]

21.\overline{c}.

And this is the firste parte of your somme.

Then for the nerte parte, multiply your firste no-
ber, that is \( \frac{102}{18} + \frac{170}{12} \) by the abatement of the seconde nomber, that is by . . . 4. \( \overline{c} \), and it will be.

\[
\begin{array}{c}
20.\overline{c}.
\hline
76.3\overline{c}.
\hline
12.\overline{c}.
\end{array}
\]

7.\overline{c}.

\[
\begin{array}{c}
7\overline{c}.
\end{array}
\]

Cr.f. As
The Arte

As by this woork ye maye see.

\[\frac{19}{5} + \frac{3}{9} = \frac{4}{20} + 76\frac{8}{12}\]

whiche being reduced to the denomination of the former number, will be tripled (sith that denominator is triple to this) and so will it be \(\frac{6}{21} + \frac{18}{18}\).

Now add those two numbers together, by puttyng their bothe numerators in one, and it will be.

\[\frac{20}{21} + 76\frac{8}{18}\]

As here appeareth in woorkke.

\[\frac{152}{60} - \frac{24}{228} = \frac{40}{36}\]

Which will not bee reduced to any smaller fraction, because the numbers be incomensurable and one of the Comike Signes is \(\frac{3}{9}\). And so is that the Comure of the multiplication.

An other wate you maye woorkke it, and all soche like, by reducyng the multiplier, into one uniforme fraction. As here in, \(\frac{14}{9}\) - 4. ye shall multyplie 4 \(\frac{2}{9}\), by \(\frac{9}{2}\), which is the former denominator, and it will be \(\frac{36}{2}\). Then putte that to 24, over the line, and set the common denominator, \(9\), under the line, and it will bee in one fraction reduced \(\frac{144}{9}\).

Scholar. Here I maye see at the firste belowe, that this fraction is an Absurde number: for the abatement after the signe, \(\frac{3}{9}\), is greater then the number befoze.
of Co$ike nombrers.

Maste. That was cõsêted befoze. But yet mae you wooke the example by it.

Scholar. That is true: and so will the numeratoz, keeyng multiplied together, make ecrately, 60. $ - 228. $ - 36. $ . As here in example of woozke, I haue set it, for my owne cæse and cæntie.

\[
\begin{array}{c}
19. $  \\
24. $  \\
\hline
456. $  \\
684. $  \\
\hline
60. $ , 228. $ , 36. $ .
\end{array}
\]

And that is the newe numerator.

And then for the seconde nomber, if the firste denôminatoz, 7 $ - 6 $ . be multiplied by the seconde denominatoz, 9 $ . it is easie seen , that thet will make, 63 $ - 54 $ . whiche shall be the newe denominatoz.

And so the interc fraction shall bee.

\[
\begin{array}{c}
60. $  \\
\hline
63. $ , 54. $ .
\end{array}
\]

That is in the smalleske nombrers and figures Co$ike.

\[
\frac{5}{13}, \frac{19}{313}, \frac{2}{14}
\]

whiche sômme, dooeth in all thynges fully agree, with the former nomber that you wroght.

Maste. Prove theim bothe by resolution: And then shal you knowe, the reason of their agremente.

Scholar. I see that the woozke of the denominatozs, dooth agree. Wherfore I will take, 3, for a roote to prove how the wozoke of the numeratozs wil agree.

And so for 19. $ , I shal haue, 5 13. And so for 3 $ ,

\[\text{Ce.v.}\]
The Arte

I shall have 9. to be added to. 5 13. And so have 3 5 22 out of which some I must abate. 5.

And then remaineth 5 17. to be multiplied by 2. 4. that is by 6 4 8. And the totall will bee (as here in woork appeareth). 3 3 5 0 1 6. which some must be abated to a smaller number, in like rate as the other was reduced, first by partition into 3. And then will it be 1 1 1 6 7 2. And again, it must be divided by 9. for that is the quantity of a square, by which the former reduction, was wrought for the Cosike signes: and then will it bee 1 2 4 0 8. And that is the first parte of the first woork. Then for the seconde parte of that woork, I shall multiply the firste numbers, that is 5 17 by the abatement of the fraction, that is by — 4 2 — 1 2.

(6th. 3. is the roote) and thereof will come — 6 2 0 4. which some I must triple, as I did his equall (that is 2 0 4. — 7 6 3. 3. — 1 2 3.) And so will it bee — 1 8 6 1 2. Now shall I adde this some, with the firste parte, which was 1 2 4 0 8. and it will bee — 1 2 4 0 8. — 1 8 6 1 2. that is, 6 2 0 4. lesse then nothing: and is the numerato of the firste woork.

Wherefore I proceed to the seconde woork, where the numerato of the fraction, being reduced to the common denominator, is 2 4. 2. — 3 6. 2. which is — 1 2 4. and in numbers resolute (keeping 3. still as a roote) it is — 3 2 4. by which if I multiply, 5 17. it will make 1 6 7 5 0 8. And that some being abated, by division into 3. and 9. as the other was, 0; els divided by 27. which is all one, it giueth 6 2 0 4. as the former woork did.

Waster. Thus I see, you are experte enough in multiplication: Wherefore I will shewe you now, the order and soyme of division.
of Coßike nombers.
Of Division.

There is noe spesiall rule to be gven, for
the woorkes of Division, other then soch-
as are all ready taughte in other woorkes
of division before. Wherefore I will by one
to 2 examples shewe you the woork of it.

The firste example of Division.

\[
\begin{array}{c}
14. \ell. - 9. \ell. \\
\hline
15. \ell.
\end{array}
\]
to be diuided by
\[
\begin{array}{c}
5. \ell. - 2. \ell. \\
\hline
3. \ell.
\end{array}
\]
doeth yeeld 42. \ell. - 27. \ell. that is in a lef-
75. \ell. - 30. \ell.
\]
for fraction by bothe reductions of nombers & lignes.

An other example.

\[
\begin{array}{c}
12. \ell. - 16. \ell. \\
\hline
2. \ell.
\end{array}
\]
divided by
\[
\begin{array}{c}
19. \ell. - 3. \ell. \\
\hline
4. \ell.
\end{array}
\]
doeth make.

\[
\begin{array}{c}
48. \ell. - 60. \ell. - 64. \ell. - 80. \ell. \\
\hline
38. \ell. - 15. \ell.
\end{array}
\]
whose nombers bee incommensurable, and therefore
made not bee reduced, but by abating one denomination Coßike. And so will it be.

\[
\begin{array}{c}
48. \ell. - 60. \ell. - 64. \ell. - 80. \ell. \\
\hline
38. \ell. - 15. \ell.
\end{array}
\]

Cc.iij. Scholar.
The Arte

Scholar. I see that you multiplye crosse wales (as in vulgare fractions) the numeratoz of the one nom-
ber, by the denominatorz of the other. And so is diuiss-
on of noe difficultie, to hym that remembreth the for-
mer rules.

Of the golden rule.

Master.

The golden rule, that is the rule of proportion, should followe now, by the commo order. But seynge there is no difficultie in it, nother any other fozme of woorke, then is in vulgare nombers, I will not staie any tyme abouthe it. Saue that for your pleasure, I haue set here certaine examples, as wel in whole nombers cosike, as in broken.

\[
\begin{align*}
32.\overline{3} & \div 6 = 4.\overline{3} \\
250.\overline{6} & \div 26.\overline{3} = 2.\overline{4}
\end{align*}
\]

\[
\begin{align*}
5.\overline{3} & \div 15 = 0.\overline{9} \\
4.\overline{5} & \div 6 = 0.\overline{7}
\end{align*}
\]

\[
\begin{align*}
31.\overline{2} & \div 61 = 0.\overline{5} \\
14.\overline{7} & \div 19 = 0.\overline{4}
\end{align*}
\]

Scholar. These fewe examples, dooe sufficiently teache the sozme of the whole rule. So that here neath noe farther explication. Wherefoze, if in this arte, there be any fozme of extraction of rootes, I praie you to procede therto.
of Cosike nombers.
Of extraction of rootes.

Master.

As in nombers Abscratte, every nomber is not a rooted nomber, but some certaine only emonget them, so in nombers Cosike, all nombers have not rootes: but as the only emonget simple Cosike nombers are rooted, whose nomber hath a roote, agreeable to the figure of his denomination.

So that. 16. C. is not a square nomber, neither hath any roote. For, although. 16. bee a square nomber, and hath. 4. for his roote, yet the denomination (whiche is. C.) hath noe square roote: but. 16. C. is a square nomber: and hath. 4. C. for his roote.

Likewases. 8. C. is a Cubike nomber, and his roote is. 2. C.; but. 8. C. hath noe roote. For, because. 8. hath no square roote, agreeable to the signe. C. nother is it a Cubike nomber, although it haue a Cubike roote, because the roote is disagreeable from the signe. C.

Scholar. I perceue that in these nombers, as well as in all other, the roote beeing multiplied by it self, will make the nomber, whose roote it is. And therefor can no nomber be called square, or Cubike, or any wales els a rooted nomber, excepte the roote of the nomber agree with his signe: Whereby I perceue well, that. 32. C. is a rooted nomber, for because that 32. hath a Surfolide roote, agreeable to the signe. So likewases. 125. C. is a rooted nomber, seyng 5, is the Cubike roote of. 125. But. 27. C. is no rooted nober.

Master. Thus you understande sufficiently, the judgemeunte of rooted nombers, and their knowledge, in simple Cosike nobers, that be utterly uncpounde. Wherefore, for extraction of their rootes, take this brief order.
The Arte

Extracte the roote of your nomber, as if it were absolute, and put to it. \( \sqrt{\cdot} \) for the denomination.

So. 27. Cubes hath for his roote. 3. \( \sqrt{3} \).

And. 49. \( \sqrt{49} \) hath. 7. \( \sqrt{\cdot} \) for his roote.

Again, the roote of 216. \( \sqrt{\cdot} \) is. 6. \( \sqrt{\cdot} \).

Scholar. This I perceive. And by like reason, the roote of. 243. \( \sqrt{\cdot} \) is. 3. \( \sqrt{\cdot} \). But why do you name numbers Cosike utterly uncompounde? For as I understande, that there bee numbers compounde, in their signes, so I see that the one have rootes also.

As. 16. \( \sqrt{16} \) hath for his roote. 2. \( \sqrt{\cdot} \). And like.

Wales. 64. \( \sqrt{64} \) hath. 2. \( \sqrt{\cdot} \) for his roote.

Master. And doe you not see, that those compounde numbers, make have moare rootes then one?

Sith. 16. \( \sqrt{16} \), hath for his square roote. 4. \( \sqrt{\cdot} \), as well as it hath. 2. \( \sqrt{\cdot} \), for his zenzizenzike roote.

So. 4. \( \sqrt{4} \) hath for his square roote. 2. \( \sqrt{\cdot} \). And hath no zenzizenzike, \( \sqrt{\cdot} \) agreeable to his whole signe.

Likewais. 9. \( \sqrt{9} \) hath no zenzicubike roote, according to his whole signe: but it hath a square roote agreeable to parte of the signe, and that is. 3 \( \sqrt{\cdot} \).

Scholar. I see that also. And so hath. 8. \( \sqrt{\cdot} \). noe zenzicubike roote, but a Cubike roote: which is. 2. \( \sqrt{\cdot} \).

Master. Therfore in compound signes, if the signe maie have soche a roote, as the nomber will yelde, it is a rooted nomber, els not.

Whereby you maie perceiue, that if any nomber compounde in signe, have a roote agreeable to his whole signe, then maie it have also, as many rootes, as ther be partes in that compound signe.

So 496 \( \sqrt{496} \) hath not onely a zenzizenzicubike roote, which is. 2. \( \sqrt{\cdot} \); but it hath a Square roote that is. 64. \( \sqrt{\cdot} \). And also it hath a Cubike roote, that is, 16. \( \sqrt{16} \). Further more, it hath a zenzizenzike roote, which is. 8. \( \sqrt{\cdot} \). And fourthly, it hath a zenzicubike roote, that is. 4. \( \sqrt{\cdot} \).

And
of Cosike nombers.

And so shall you judge, of all other like.

Scholar. This shall suffice, as I will practise the mater, at moare leisfer. But and if the nombers bee compounde, with signes of addition, is there then any speciall order for their rootes: As in this example.
8 1.2.3-—= 27.4, where I have made eche parte to be a rooted nomber.

Master. In deede. 8 1 2 3- hath bothe a Square roote, and also a zenzizezike roote. But 27 4 hath none of those twoo rootes, although it haue a Cubike roote, which the other nomber wanteth. And therefore is not that whole nomber, a rooted nomber.

But to the intente, that you maie be the moe certaine of rooted nombers, I will tell you certaine notes, how it maie be knowne, whether your nomber be a rooted nomber.

Firste, if the nomber annered to the greatest signe of that compounde Cosike nomber, be not a rooted nomber, the whole nomber can not be a rooted nomber.

Secondarily, if the nomber that is lowned with the leaste Cosike signe, be not a rooted nomber, the whole nomber can not be a rooted nomber.

And eche of these bothe rootes (if thei haue any) are partes of the whole roote, for the compounde Cosike nomber.

Thirdly, if the nomber be a rooted nomber, every parte of it, that is not a rooted nomber, is a meane nomber, betwene the greatest and the leaste.

Fourthly, if 2 be any denomination in it, then is 2 an other denomination in it also.

Fiftly, and generally, all rooted nobers, other are specially framed, by orderly multiplication, 02 els are nombers equalle to some one rooted nomber Abstract.

Now specially framed are soche, as are made by multiplicatio of one nomber by it self, and litle 02 no- thyng altered from that very forme.

Dr. i. Example
The Arte

Of square roots.

Example of \( 529 \sqrt{\text{c}} - 184 \sqrt{\text{c}} - 16 \sqrt{\text{c}} \), which is a square number, made by multiplication of \( 23 \text{c} + 4 \sqrt{\text{c}} \), by itself. This number made have his roote orderly extracted thus.

\[
529 \sqrt{\text{c}} - 184 \sqrt{\text{c}} - 16 \sqrt{\text{c}} = (23 \text{c} + 4 \sqrt{\text{c}})^2
\]

In the firste number, I finde the square roote to bee \( 23 \). And for his denomination, I take halfe the Cosike signe \( 23 \), and that is, \( \sqrt{\text{c}} \). For as \( \sqrt{\text{c}} \) multiplied by \( \sqrt{\text{c}} \) doeth make \( 23 \). So in division by \( 2 \) and in extraction of square rootes, I shall take the \( \sqrt{\text{c}} \) for the halfe of \( 23 \) and the denomination of his roote: and so set it done in the quotiente.

Then I shall double the number Abstratte of that quotiente (keeping his Cosike signe unaltered) and that double shall I set euermore under the nerte number, toward the righte hande. As here, you see, I haue set \( 46 \) (whiche is the double of \( 23 \)) with his signe \( \sqrt{\text{c}} \) under the seconde number. And there I perceive I maie haue it. 4. tymes, if I doe diuide (as I ought) \( 184 \) by \( 46 \). And that, 4. I sette in the quotiente, with the signe \( \sqrt{\text{c}} \); being \( 26 \sqrt{\text{c}} \), diuided by \( \sqrt{\text{c}} \) doeth yelde \( \sqrt{\text{c}} \).

Lasse of all, I muste multiplye that parte of the quotiente, \( 4 \sqrt{\text{c}} \), by itself, and it will yelde \( 16 \sqrt{\text{c}} \), which being substracted also (as it should) leaueth nothyng remaining of the square number.

This order must you kepe in all square nombers, how great so euer they be. As in this seconde exemple.

\[
25 \text{c} - 80 \sqrt{\text{c}} + 144 \text{c} - 81 \sqrt{\text{c}} = 9 \sqrt{\text{c}}
\]

\[
5 \text{c} + 64 \sqrt{\text{c}} - 10 \sqrt{\text{c}} - 16 \sqrt{\text{c}} = 9 \sqrt{\text{c}}
\]

The
of Coßike nombers.

The roote of the first number is \(5\), whiche I set in a quotiente.

Then doe I double that, \(5\), and it maketh, \(10\), to be sette under \(8\), with his denomination, whiche is \(8\), like to the roote.

That \(10\), maie be founde in \(8\), \(8\), times, \(8\), therfore I set \(8\), in the quotiente, with the signe \(-\) and the denomination \(8\),. And then doe I multyple that \(8\), squarely, whiche giueth, \(= 64\), to be subtracted out of \(= 26\), and so remaineth \(9\).

After this I double all the quotiente again, where of commeth \(- 10\), \(= 16\). And because there is a remainer, over the number that I wrought lasse, I must set \(10\) under the remainer, and the other number in order, as you see it set.

Then seke I how often tymes maie, \(10\), divide \(9\), and I finde the quotiente to be \(9\). And likewise \(- 16\), multiplied by \(9\), doeth make \(= 144\), equalle to the somme over it. And so subtracteth it cleane. Therfore to ende that woork, I multyple the lasse quotiente, by it self square, and it yeldeth \(= 81\), which is to bee subtracted out of the like somme, in the square number: and so resteth nothing. Therfore I justly afirme, that the firste number is a square number, and hath for his roote, \(5\), \(= 8\), \(= 9\).

Scholar. That maie I sone proue, if I multyple

\[
\begin{align*}
5\cdot \cdot & + 8\cdot \cdot \cdot \cdot = 9\cdot \cdot \cdot \\
5\cdot \cdot & + 8\cdot \cdot \cdot \cdot = 9\cdot \cdot \cdot \\
25\cdot \cdot & + 40\cdot \cdot \cdot \cdot = 45\cdot \cdot \cdot \cdot \\
\quad & + 40\cdot \cdot \cdot \cdot = 64\cdot \cdot \cdot \cdot \\
81\cdot \cdot & + 72\cdot \cdot \cdot \cdot = 45\cdot \cdot \cdot \cdot \\
\quad & - 72\cdot \cdot \cdot \cdot \\
25\cdot \cdot & + 80\cdot \cdot \cdot \cdot = 26\cdot \cdot \cdot \cdot = 144\cdot \cdot + 81\cdot \cdot \cdot \cdot \\
\end{align*}
\]

That
The Arte

that roote by it self, as here I have done it. Whereby I have not onely confirmed it to be a square nombre: but also I have espiied, that you used the nombre not so plainly set doune, as the particulare multiplicati: on did make it: but rather as a reasonable reduction would expressse it. I meane in the 3.3., where the particulare multiplication hath 64.3.3., and 90 3.3. For whiche twoo nombres you sette one, that resulteth of the bothe, that is 26 3.3.

Mater. But if you would take the nöber in that softe, the twoo ke would be not onely all one: but also somewhat plainer to bee perceived of a learner. And therefore for your pleasure, I will set sozke here, the example of that woozke. And loe, here it is.

\[
\begin{align*}
25 \cdot 3 &+ 80 \cdot 3 + 64 \cdot 3 &- 90 \cdot 3 &- 144 \cdot 3 + 81 \cdot 3 + 81 &- 9 \cdot 3 \\
5 \cdot 3 &+ 10 \cdot 3 + 64 \cdot 3 &- 10 \cdot 3 &- 16 &
\end{align*}
\]

Scholar. By comparynge these bothe formes of woozke together, I doe best understande, the reason of the firste woozke.

Mater. onc example moare of this kinde of extraction of rootes, will I set doune, that maie be a generalle patroné, for all the varieties, in this sozte of rooted nombres. And if you examine it diligently, and marke it well, you shall neade fewe other examples, for this kinde of square nombres.

The Square nombre, with the woozke of extraction of his roote so, loweth here.
The Arte

that roote by it self, as here I have done it. Whereby I have not onely confirmed it to be a square nomber: but also I have espiet, that you bscd the nomber not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would exprass it. I meane in the $3^2$, where the particulare multiplication hath $3 \times 3 = 9$. For whiche twoo nombers you sette one, that resulteth of the bothe, thast is $263^2$.

Muster. But if you would take the nober in that sorte, the twooke would be not onely all one: but also somewhat plainer to bee perceived of a learner. And therefore for your pleasure, I will set for the here, the example of that woorke. And loe, here it is.

\[
25\frac{3}{4} + 80\frac{5}{6} + 64\frac{3}{2} - 9\frac{3}{8} + 14\frac{3}{4} + 8\frac{5}{6} - 9\frac{3}{8} + 5 + 10 + 64 - 10 - 16.
\]

Scholar. By comparyng these bothe formas of woorke together, I dooe better understande, the reason of the firste woorke.

Muster. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a gerneralle patrone, for all the variettes, in this sorte of rooted nombers. And if you examine it diligently, and marke it well, you shall neade fewe other examples, for this kinde of square nombers.

The Square nomber, with the woorke of extraction of his roote foloweth here.
The square number, with the woorke of extraction of his roote.

\[
\begin{array}{c}
36 \times 3 = 108 \\
36 \times 8 = 288 \\
36 \times 25 = 900
\end{array}
\]

The Roote.

\[
6\sqrt{5} - 5.35 - 4.0 - 3.5 - 2.2 - 1.9.
\]

The proof by Multiplication.

\[
\begin{array}{cccccccc}
6.5 & \times & 5.35 & = & 32 & \times & 3.5 & = & 2.2 & \times & 1.9 \\
6.5 & \times & 5.35 & = & 32 & \times & 3.5 & = & 2.2 & \times & 1.9 \\
36.5 & \times & 3 & = & 108 & \times & 1.5 & = & 45 & \times & 1.5 \\
36.5 & \times & 3 & = & 108 & \times & 1.5 & = & 45 & \times & 1.5 \\
24.5 & \times & 2 & = & 96 & \times & 0.5 & = & 24 & \times & 0.5 \\
24.5 & \times & 2 & = & 96 & \times & 0.5 & = & 24 & \times & 0.5 \\
18.5 & \times & 1 & = & 12 & \times & 0.5 & = & 6 & \times & 0.5 \\
18.5 & \times & 1 & = & 12 & \times & 0.5 & = & 6 & \times & 0.5 \\
12.5 & \times & 0.5 & = & 6 & \times & 0.5 & = & 3 & \times & 0.5 \\
12.5 & \times & 0.5 & = & 6 & \times & 0.5 & = & 3 & \times & 0.5 \\
6.5 & \times & 0.5 & & & & & & & & \\
6.5 & \times & 0.5 & & & & & & & & \\
3.5 & \times & 0.5 & & & & & & & & \\
3.5 & \times & 0.5 & & & & & & & & \\
2.2 & \times & 0.5 & & & & & & & & \\
2.2 & \times & 0.5 & & & & & & & & \\
1.9 & \times & 0.5 & & & & & & & & \\
1.9 & \times & 0.5 & & & & & & & & \\
\end{array}
\]

Scholar. It maie appeare easily, that this example forthweth many others, it dooth contain so many varietie of signes Cosike, multiplied to diversely.

And in this number also, as well in the other, I see that many numbers be omitted, or reduction; namely in the third, fourth, five, and sixth orders of numbers. For in the second orders, and in the third, there is no varietie of the signes and

Wherefore to see the varietie of woork, I will sette downe the numbers, as they rise in particular multiplication, and in it I will make an experimente of my cunning. As here followeth.

\[
\begin{array}{cccccccc}
36 & \times & 5 & = & 180 & \times & 1 & = & 180 \\
36 & \times & 8 & = & 288 & \times & 2 & = & 576 \\
36 & \times & 25 & = & 900 & \times & 4 & = & 3600
\end{array}
\]

\[
\begin{array}{cccccccc}
36 & \times & 10 & = & 360 & \times & 2 & = & 720 \\
36 & \times & 6 & = & 216 & \times & 3 & = & 648 \\
\end{array}
\]

Db. 19. Where
of Coßike nombers.

Where fo, wyne owne ease, and aied of memozie, I have set under every doublyng of the quotiente: And the somme that amounteth, by the multiplication of the same, into the newe quotiente, with the Square of the same newe quotiente.

Whereby I perceiue that the nombers, doe not go in soche omer, that every odde place, maketh a newe roote, as it dooth in nombers Abstrakte. But sometime I must take 2. places nexte together, and at an other tyme, I shall skippe 2.023. places.

Walter. You marke it well. And yet that is a good and true rule, that some menne teache: that in these Coßike nombers, as well as in other Abstrakte nombers, you shall marke every odde place, and under ech of them to finds a Square roote. But that is to be understant, when the nombers are sette, in their brekette and exactette omer.

These sewe examples may suffice, for a declaratid of extracynng the roote of Square nombers, made by The roots of multiplication. And now touchynng those nombers, nombers equal that bee equalle to some rooted nomber, and namely to besquarts. Sonic as be equalle to a Square nomber, I will teache you how their roote maie be extracted.

But firste you shall marke, that a Square byeing compared, as equalle to rootes and nombers, the rootes maie be coupled with thenombers onely, in 3. formes. That is, \( z \). \( = \) \( 9 \) (which is all one with \( 9 \). \( = \) \( z \)) ozels thus, \( 9 \). \( = \) \( z \). D: thirdly, \( z \). \( = \) \( 9 \). And soo eche of these. 3. fortes, there is some varietie, in the extraction of the roote. And in them all onche agremente.

For the first forme, where \( z \). \( = \) \( 9 \) is equalle to The firste \( z \) take these exemples 1 \( z \) is equal to \( 4 \). \( z \). \( = \) \( 21 \) forme. 02. 1 \( z \) is equal to 35. \( = \) \( 2 \). \( z \). Likewise 1 \( z \) is equal to 10 \( z \). \( = \) \( 75 \). 02. 1 \( z \) is equal to 105. \( = \) \( 8 \). \( z \).
of Cosike nombers.

Wherefo, wnte owne ease, and aied of memozie, I haue set vnder vvery doublyng of the quotiente: And the somme that amounteth, by the multiplication of thesame, into the newe quotiente, with the Square of thesame newe quotiente.

Whereby I perceiue that the nombers, doe not go in soche ozer, that every odde place, make th a newe roote, as it dooth in nombers Absraete. But sometime I must take 2. places neste together, and at an other tyme, I shal scippe 2. or 3. places.

Watter. You marke it well. And yet that is a good and true rule, that some menne teache: that in these Cosike nombers, as well as in other Absraete nobers, you shall marke every odde place, and under sche of them to finds a Square roote. But that is to be understande, when the nombers are sette, in their breseste and exacteste ozer.

These seve examples maie suffise, for a declaratio of extractyn the roote of Square nombers, made by The rootes of multiplication. And now touchynge these nombers, nobers equal that bee euall to some rooted nomber, and namely to besquarts. soche as be euall to a square nomber, I will teache you how their roote maie be extracted.

But firste you shall marke, that a Square beynge compared, as euall to rootes and nombers, the rootes maie be coupled with the nombers onely, in 3. fomese. That is. 2. — — 9 (whiche is all one with 9. — — 2.) 02 els thus. 9. — — . 2. D. thirdly, 2. — — 9. And soe eche of these. 3. foertes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the firste fomse, where 2. — — 9 is euall to 1. 2. take these eases aeplis 1 2. is euall to 4. 2. — — 21 2. forme. 02. 1 2. is euall to 35. 9. — — 2. 2. Likewise 1 2. is euall to 10 2. — — 75. 9. 02. 1. 2. is euall to 105. 9. — — 8. 2.
The Arte

In all these examples, and other so like, you must first consider the number annexed with the signe \( \times \), (which is the middle quantity) and the halfe of it shall you note, for with it shall you workke twice. First you shall multiplye halfe of that number by it self, and this is the firste worke, and to it shall you adde the other whole number, that is joyned with \( \div \). And ther will ever more make a square number: out of whiche square you shall extracte the roote. And to that roote shall you adde halfe the number, that was annexed with the signe of \( \times \), (which was the number that I have you to mark). And this is the seconde worke. The total that commeth of this addition, is the roote of the compounde Co"ake number.

An example

Example of the firste. \( 4 \times 2 \div 2 \times 2 = 1 \). halfe the number annexed with \( \times \). Is 2. Whole Square is 4. that shall I put to. 2 1. and there riseth. 2 5. beynge a square number, and haungg 5. for his roote. To that 5. I joyn thalfe the number annexed with \( \times \), and it maketh. 7. Which is the number that I seke for; and is the roote to. \( 4 \times 2 \div 2 \times 2 = 1 \).

The proofe.

For triall whereof take. 4. rootes, that is. 2 8. and putte to it. 2 1. and thereof commeth. 4 9. which is a square number, and hath. 7. for his roote.

An other example.

Scholar. Then can I doe the like with the second example. 35. 2 \( \div 2 \times 2 = 1 \). And firste the halfe of. 2. is 1. and the Square of it is 1. which I put to. 35. and it maketh. 36. a Square number; whose roote is 6. To that 6. if I adde 1. that was the halfe before reserved, it will make. 7. Which is the roote that I doke seke.

The proofe.

The proofe is this; 2. rootes maketh. 1 4. and. 3 5. gi-

neth. 4 9. whose roote is 7.

The third example.

Likewise for the thirde example. \( 10 \times 2 \div 2 \times 2 = 75 \)

I woorkke thus. Halfe. 1 0. is 5. and his Square is 2 5. that doe I adde to. 7 5. and there riseth. 1 0 0. Whole roote is. 1 0. to whiche roote I adde 5. and there com-
of Cobike numbers.

meth. 15. that is the roote whiche I would have.

And that I make prooe by triall in this sorte. 1 0.
rootes giue. 150. unto whiche if I add. 75. there will
amounte. 2 2 5. whiche is a Square number: and hath
1 5. for his roote.

The fourthe example is. 1 0 5. 9 — 8. z 0. where The fourthe
I take strike the halfe of. 8. that is. 4. and it in Square example.

giveneth. 1 6. whiche I adde to. 1 0 5. and there amount-
teth. 1 2 1. beyng a Square number, and the roote of it
1 1. unto whiche I shall adde. 4. for halfe the number
of rootes: and so there riseth. 1 5. as the roote that I
seek for. And to approoe if I take. 8. time. 1 5. whiche? The prooe.
is. 1 2 0. and add it unto. 1 0 5. and so commeth. 2 2 5.
For the square, and the roote of it is. 1 5.

Walter. The like order of woike shal you use, in other for:
all numbers Cobike compound, whiche any 2. numbers mes in like
with immediate denomination Cobike, are equaile to sorte.
one of the nexte denomination, in order above them.

As. 1 0 5. is equaile to. 3. 3 2. — 1 0. 0 0.

And again. 1. 5 7. equaile to. 6. 3. 3 2. — 1 — 4 0. 0 0.
Liekewise. 1. 5 7 2. equaile to. 3. 3 2. — 1 — 2 8. 3 2. 3 2.

But in all these the roote shal beare name of the great-
ter quantie.

Scholar. By the former order of woike, I shal in The firs-
the firste of these. 3. examples, take halfe. 3. (because it
is the number of the middell quantite). And that is ¼.
and that shal I multiplye squarely, and so will there
rise ¼. unto whiche I shall add. 1 0 0. 0 0. 2 0. 2. And that ma-
kethe 2 0. whiche is a Square number, and his roote is 2 0.
unto whiche I must put the firste halfe, that is ¼, and
then will it be ¼, or els. 5. whiche is the Cubike roote of
that number. 3. 3 7. — 1 — 1 0. 0 0. beyng equaile to 1 0 0.

For prooe whereof, I multiplye. 5. Cubikely, and it The prooe.
maketh. 1 2 5. Then doe I multiplye it squarely, and it
will be. 2 5. Pow. 3. 3 2. is. 7 5. and. 1 0. 0 0. maketh. 5 0.
whiche bothe added together, giue. 1 2 5.
The Arte

The seconde example. In the seconde example, where \( \sqrt[3]{2} \) is equalle to \( \sqrt[3]{6} \). I shall take halfe \( \frac{6}{2} \) (which is the number of the middell quantitie) and that is \( 3 \). and the square of it is \( 9 \), which I must adde unto \( 40 \) and thereof commeth \( 49 \), which is a square number and hath, or his roote, unto which I adde \( 3 \), and so have \( 10 \) for the Surculide roote, of \( 6 \).

The proofe. And so proofe I saie, if \( 1 \) see the roote, then is \( 10 \), the square, \( 4 \), the Cube, the \( 3 \) is \( 10000 \). And the Surculide, \( 10000 \), wherefor, \( 6 \) make \( 60000 \) and \( 40 \) whole, \( 40000 \). And bothe thei together doe make \( 100000 \) which is the quantitie of the Surculide.

The thirde example. In the thirde example, \( 1 \) is equalle to \( \sqrt[3]{28} \). whole Zenzicubike roote, I see in this sorte.

The proofe. Firstly I take halfe \( 3 \) (as the number of the middell quantitie) that is \( \frac{3}{2} \) that maketh in square \( \frac{3}{2} \) which I adde unto \( 28 \) (that maketh \( \frac{28}{2} \)) and it yeldeth \( \frac{28}{3} \) which is a square number, and his roote is \( \frac{4}{3} \) unto which I adde \( 3 \), and it will be \( \frac{40}{3} \). which is the Zenzicubike roote unto the foregoing number, \( \sqrt[3]{28} \).

A thirde forme. Matter. Yet one other forme is there, that in all things saue in one poynete onely followeth the same rule. And that is whe the 3 denominations doe not go immediatly together, but yet are equally distance. As \( \sqrt[3]{2} \) and \( \sqrt[3]{9} \). where the distance is one onely quantitie. Likwais \( \sqrt[3]{2} \), \( \sqrt[3]{4} \), and \( \sqrt[3]{9} \) which differ by 2 quantities. And in like sorte \( \sqrt[3]{6} \), \( \sqrt[3]{7} \) and \( \sqrt[3]{5} \) are distance by 3 quantities. And so of other, how many so cuer bee omitted, so that the difference bee equalle
of Cosike nombers.

equall. In all whiche you shall worke, as you did in the former rule, till you have canded all that worke. But then have you here, one thing moze to bee con-
sidered. For the laste nomber, whiche you have founde, is not the roote, but a rooted quantitic: And his roote is the roote that you seke for.

Scholar. Doe you meane the square roote of that quantitic, or some other?

Maste. It maie be any kinde of roote, in diverse nombers, but not in one nomber. Wherefore for your certeintie marke this rule.

If the denominations of your nombers, differ one-
y by one, then is it a square nober, that you doe finde by the practice of the laste rule. And therefore shall you take his square roote, for the roote of your nomber.

But if the denominations differ by 2. quantitics, then shall you extracete a Cubike roote, out of your laste nomber. And if the distance become 3. quantities, the roote must bee a zenzizenzike roote: and so 4. quantitics distance, a Surfolide roote, and so for the.

As for example. 1. 2. 3. is equalle to 8 0. 3. An example 2 0 0 0. 9. Now for to finde the roote of 8 0. 3. 2 0 0 0. 9. I worke thus. Firste I take the halfe of 8 0. (because it is the number of the middle quantitic) and that halfe is. 4 0. Which I multiple Square, and it makeseth 1 6 0 0. To it I adde 2 0 0 0. and it will bee 3 6 0 0. Which is a square nomber, 4 6 0. is his roote: to that. 6 0. I shall adde the foresaided 4 0. and then will it bee 1 0 0. Which nomber in the firste rule, had been the true roote. But here consideryng the distance is of one quantitic, I must extracete his square roote, which is 1 0. And that is the zenzizenzike roote, that my nomber containeth.

An other example. 1. 2. is equalle to 4 0 0. 2. The seconde — 5 7 3 4 4. 9. I take 2 0 0. for the halfe of the midi example. dwell quantitics nomber, and multiplying it square, I
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finde. 40000. which I put to. 57344. and then I have. 97344. which is a square number, and his roote is 312 unto which I shall adde the halfe of 400 and so will it bee. 512. But now must I take the cubike roote of this number (that is. 8) for my roote, that I desire. Because the denominations in the number, differ by 2 quantities.

Scholar. I see very well the order of this woxke: And the proosfe is in like sozte, which I maie practise by my self at any tyme. Wherefore I praie you, pro- cede sozthe to other rules.

Mater. This is sufficiente for the firste sozte. Now for the seconde sozte, in numbers diminute oz re; fidualle. Where, 2, is equalle to. 9 — 72 , the soune of woxke is like unto the other, in all points faine in one. For in feds of the laste addition, you shall bse in these numbers, Subtraction. As here for example, when I sale. 12 , is equalle to. 609-4. 72 , to finde the roote, firste I take the halfe of. 4. (because it is the number of the middell signe) and that halfe byeng. 2. doeth make in square. 4. which I put to 60 and so is it. 64. a square number, and hath. 8. for his roote. From which roote (by the order of this rule) I must abate. 2. that is the halfe of the firste number of rootes. And then will there remaine. 60; for the verie roote of. 609-4. 72 , byeng equalle to. 12.

The proosfe. Scholar. That is done proved. For. 6. byeng the roote, then. 4. 72 , maketh. 2. 4. which byeng abated out of. 60. leaueth 36 and that is the tulle square unto. 6. as the equation saith.

The seconde example. Mater. An other example is this. 12 , is equalle to. 162. 72 , — 9. 72 , 72 .

Scholar. That can I woxke, thus: Firste I take the halfe of. 9. (because it is the number of the middell signe) and it is ½ , which I multiplie squarely, and it will be ¾ , that must bee added to. 162. 07 44 , and then will
of Cosike nombers.

Will there amounte $\frac{11}{4}$, which is a square number, and hath for his roote $\frac{1}{4}$ out of whiche, by this rule, I must abate $\frac{1}{4}$, and then riseth $\frac{1}{4}$, that is, 9, which is the very roote to 162. Then $9\cdot \frac{1}{2}$, being equall to $1\frac{1}{4}$.

And for the proofe, I multiplye 9. Cubikely, and it The proofe.

geth $1\frac{1}{4}$, so that at 162. I. doc make. 118098.out

of whiche I must abate. $9\cdot \frac{1}{2}$, that is, 59049. (by the same roote, lith. $1\frac{1}{4}$, is. 6561). And then will there remaine 59049, whiche is the luste quantitie of $1\frac{1}{4}$.

Waller. Yet one example moxe shall you have of The third.

example.

When $1\frac{1}{2}$ is equalle to $275456$. $\frac{1}{2}$

I demaunde of you, what is the valewe of $1\frac{1}{2}$?

Scholar. I searche it thus. The nomber of the midkel sike is 26. whose halfe I must takke, and first multiplye it squarely, and there will rise 169, whiche I adde to 275456, and it will be. 275625, whiche is a square number, and hath for his roote. 525. from whiche number I must abate halfe the number, of the midkel sike, that is. 13. and so the there will remaine

512. whose Cubike roote I must extract, because the denominations differ by. 2. quantities, and that roote will be.8, whiche is the Cubike roote to 512, but to the number propounded, it is the zenzi cubike roote.

Waller. This is quantie to the worke of the seconde softe. Now for the third softe of equation, The thirde where. $\frac{1}{2}$ is equalle to $\frac{1}{2}$.

I will givne you sorte of equal

a braise admonition onely, though it differ from bothe numbers, the other. 2. rules, in some of woorke. For as the eqaulitie may be in diverse softe, so some tymes you maye use the woorke of the firste softe, by Addition of halfe the number of the middle sike, and some times you shall woorke by subtraction. Wherein this is the difference, from the seconde rule. That there you doe

Eg. 9. subtracte
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Subtract halfe the number of the middell signe, from the roote whiche you suonde. And in this thirde rule, you shall subtracte the roote from the halfe, and not the halfe from the roote. For because that that roote, is ever lesser then that halfe.

And in this rule, this is specially to bee observed: that the Square of halfe the number, of the middell signe, will ever more bee greater, then the number of the lesser signe: And therefore shall the number of the lesser signe, bee abated out of that square. And the remainder will bee a Square number, with whiche you shall wootke, as I have taught you before.

And farther in this rule, it is commonly seen, that every soche equalle number, hath 2. valuations for his roote, I meaneth that any of those 2. numbers, will bee as the roote in this equation. For otherwise no number can have 2. rootes of one denomination.

Scholar. I understande you thus. That no number can have 2. square rootes, or 2. cubike rootes, and so forth: Else one number may have 3. or 4. rootes. As 64 hath 8, for his Square root 4, for his cubike root 2, and 2 for his sencon cubike root.

Master. You take it well. And farther for the easie knowledge of those 2. numbers, or rootes: They must bee soche, as beeing added together, will make the number of the middell signe: and beeing multiplied together, will make the number of the least signe. And so may you finde them without farther multiplication, or extraction of rootes.

For example I sette firste 15. equalle to 16.79.

63 3 where I maie espie quickly, that 63 can have no more partes to his composition, but 3. 7. 9. 21 And if I take 3. and 21. then their addition will bee greater then 16. but 7. and 9. maketh inste 16. by addition, and 63. by multiplication. And therefore thei shall be the 2. rootes.

Scholar.
of Cosike nombers.

Scholar. I will proove that by examination, thus. If 7 be the roote, then is 49 the square. And 16 - 7 make 112 out of whiche I must abate 63 and there retest. 49 equalle with the square: so is that a true roote. Then so is his square is 81. And 16 - 7 do make 1 4 4 frome whiche I shall abate 63, and the remain-ner will be 81 equalle to the square. And so is that also a true roote.

Master. How wooke it by the other rules, that I taught you.

Scholar. Firste I take 8, as halfe the number of the middell signe, and that multiplied Square, doeth give 64 frome whiche I call abate 63 and then doeth there remain but 1, whiche is counteed as a square number, and his roote to be 1 also, whiche if I adde to 9 it will make 9, that is one of the rootes: And if I abate it from 8 it will leave 7, whiche is the other roote. And thus I see one wooke costrmeth the other.

Master. Take this for the seconde example. 1 3 2 5 5 is equalle to 8 7 2 1 2, 3 5 2 8 8. what is the roote faic you?

Scholar. To finde it, firste I loke for the partes of 1 2. And the be 2, 3, 4, 6, of whiche 2, and 6, does serve my purpose, for their addition maketh 8, and so doeth not 3, and 4. Wherefore I saie, that 2, maie bee the roote, and so maie 6. But for farther triall of it: I wooke it by the other rule, saynng halfe 8 is 4, and his square is 16. From whiche I abate 1 2, and there remaineth 4, whose roote is 2, that I maie adde to 4 and so haue 8. 6 for one roote: 0; els abating it from 4. I shall haue 2, for the other roote.

The proove is manifeste for 6, beeing a roote, the xenzicube is 4 6 6 5 6. The Surfolide is 7 7 7 6. And the xenzicenzike is 1 2 9 6. So that 8 7 2 5 2, do make 6 2 2 0 8 And 1 2, 3 5 2, are 1 5 5 5 2, whiche being abated out of 6 2 2 0 8 do leaue 4 6 6 5 6, the true quantitie of 1 5 2. And
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And so is that woork good, 6. byynge a roote.

How if. 2. be settte for a roote: then is the. z z 16. the z z 32. and the z z 64. And so are 8. z z equall to. 2 5 6. And 1 2. z z. pellde. 1 9 2. Wheresofe abating 1 9 2. out of. 2 5 6. therre resteth. 6 4. the tylle quantitie of. 1. z z. And so is that woork also good. and 2. a true roote.

Master. Now prove this thirde exemplar. where 1. z z 3 is equall to. 2 0 0 0. z z 47 0 0 1 6 29.

Scholar. Halfe the numbere of the middel signe is 1 0 0 0. And the square of it is. 1 0 0 0 0 0 0. From whiche I shall abate. 4 7 0 0 1 6. and therre will remaine 5 2 9 9 8 4. whosse square roote by triall of extraction. I finde to be 7 2 8. whiche I maie other adde to 1 0 0 0 and so therre riseth. 1 7 2 8. whiche I finde to bee (as it ought) a cubike number. And his roote to be. 1 2.

But and if I abate 7 2 8. from 1 0 0 0. therre will remaine 2 7 2. whiche is no cubike number.

Master. So that here semeth to be but one roote.
And yet these 2. numberes. 1 7 2 8. and 2 7 2. kepe soche a rate, that byynge multiplied together. thei make 4 7 0 0 1 6. whiche is one of the numberes. and byynge added together. thei make 2 0 0 0. whiche is the other number of the same cubike residuall.

But now prove in other like numberes. whiche have some diumtance. betweene their denominations. whether it will so happen still. As namely in this. where 1. z z 3 is equall to. 1 2. z z 4 7 2 9.

Scholar. Halfe. 1 2. is. 6. and his square. 3 6. from whiche abating. 3 2. therre is lefte. 4. whose roote is. 2. And if I adde that. 2. to. 6. it maketh. 8. whiche is a cubike number. and hath. 2. fez. his roote. But if I abate 2. from. 6. therre remaineth. 4. whiche is no cubike number. and therfore hath no locke roote. And yet these 2. numberes. 4. and. 8. by addition. make the middell number. and by multiplication. thei make the laste number.
of Cosike nombers.

Master. Prove ye yet ones agaynce in a nomber, The fift where one quantite onely is omitted. As when 1. 5. example. is equal to 24. T. = 135. T.

Scholar. 12. maketh in square. 144. from whiche I shall deduce 135. and then resteth 9. whose square roote is 3. whiche if I adde to 12. it will bec. 15. and hath no square roote, as here is required. But if I a-bate 3. from 12. then remaineth 9 whose square roote is 3. and sreuth to the nomber, as I have here produc'd in my tables. And. 9. and. 15. kepe the customable rate. For by addition thei make 24. And by multiplication, thei yelde. 135.

But in all these examples, where the denominations be are a distance, I can finde but one roote, and not 2. As it was in the other exaples of the same rule.

And in some of theim, the greater nomber containeth the roote: but in other, the lesser nomber hath the roote.

Master. Bicause I can not stale now, about this varietie, I will remitte it till another tymc. But this by the wate, I must admonishe you, that I doe folowe here, the common forme of writers, in calling these rootes, that rise in equatio, where as thei are not the rootes of those nombers, but are the value of a roote. For of a Cosike nomber, the roote must neades bee a Cosike nomber also. And soche as by multiplication will make the rooted nomber: But so can not those nombers doe.

And here will I make an ende, of the workes of Cosike nombers. And now will I apply them to practive in the rule of equation, that is commonly called Al- gebers rule.
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The rule of equation, commonly called Algebrers Rule.

Therefore I taught you, the common forms of worke, in numbers Denominable, which rules are used also in numbers Abstracte, & like waies in Surde numbers. Although the forms of these workes be severalle, in every kind of number. But now will I teach you that rule, that is the principall in Cosike workes: and so which all the other doose.

This Rule is called the Rule of Algebr, after the name of the inuentoure, as some men thinke: or by a name of singular excellencie, as other judge. But of his use it is rightly called, the rule of equation: because that by equation of numbers, it doeth dissolve doubtfull questions: And unsolve intricate rules. And this is the order of it.

The somme of the rule of equation:

When any question is propound, apperteyning to this rule, you shall imagin a name for the number, that is to bee sough in the number, that you learned in the rule of false position. And with that number shall you procede, according to the question, until you finde a Cosike number, equal to that number, that the question expresseth, which you shall reduce
of CoBike nombers.
reduce ever more to the leaftest nombers. And then
divide the nomber of the lesser denomination, by
the nomber of the greateste denomination, and
the quotient doeth aunswer to the question. Ex-
cept the greater denomination, doe beare the signe
of some rooted nomber. For then must you extract
the roote of that quotiente, according to that
signe of denomination.

Scholar. It semeth that this rule is all one, with
the rule of false position: and therefore mighte so bee
called: seyng it taketh a false nomber, to worke with al.

Master. This rule doeth farre excell that other.
And dooth not take a false nomber, but a true nom-
ber for his position, as it shall bee declared anon.
Wherby it maie bee thoughte, to bee a rule of won-
dersfull invention, that teacheth a manke at the firste
worde, to name a true nomber, before he knoweth re-
solutely, what he hath named.

But because that name is common to many nom-
bers (although not in one question) and therefore the
name is obscure, till the worke doe detect it, I thinke
this rule might well bee called, the rule of darke posi-
tion, or of strange position: but not of false position.

And so, the more easie and apte worke in this arte
were dooc commonly name that darke position. i.e.
And with it doe we worke, as the question intendeth,
till we come to the equation.

This rule of equation, is divided by some men, into
diverse partes. As namely Schenbelius dooth make. 3.
The partes of the rule.

rules of it. And in the seconde rule, he putteth, 3. sece-
ralle cannots. Some other men make a greater nomber
of distinctiones in this rule. But I intende (as I thinke
beste so, this treatise, whiche maie serue as farre
FF. i. as
as their workes doe extende) to distincke it onely into twoo partes. Whereof the firste is, when onenumber is equalle vnto one other. And the seconde is, when one number is compared as equalle vnto 2. other numbers.

Alwaies wyllyng you to remembre, that you reduce your numbers, to their leaste denominations, and smalleste formes, before you procede any farther.

And again, if your equation be soche, that the greatesse denomination Cosike, be joined to any parte of a compounde number, you shall tourne it so, that the number of the greatesse signe alone, male stande as equalle to the reste.

And this is all that needeth to be taughte, concernyng this woorkes.

Howbeit, for easie alteratiō of equations. I will propounde a fewe examples, because the extraction of their rootes, maie the more aptly bee wroughte. And to a-vuide the tedious repetition of these woordes: is equalle to: I will sette as I doe often in woorkes binse, a paire of paralleles, o2 Gemowe lines of one lengthe, thus:———, because noe. 2. thynges, can be more equalle. And now marke these numbers.

1. \[14.\overline{2}e\] — \[15.\overline{9}e\] — 71.\overline{9}e
2. \[20.\overline{2}e\] — \[18.\overline{9}e\] — 102.\overline{9}e
3. \[26.\overline{2}e\] — \[10.\overline{2}e\] — \[9.\overline{2}e\] — \[10.\overline{2}e\] — 213.\overline{9}e
4. \[19.\overline{2}e\] — \[192.\overline{9}e\] — \[10.\overline{2}e\] — \[108.\overline{9}e\] — \[19.\overline{2}e\]
5. \[18.\overline{2}e\] — \[24.\overline{9}e\] — \[8.\overline{2}e\] — \[2.\overline{2}e\]
6. \[34.\overline{2}e\] — \[12.\overline{2}e\] — \[40.\overline{2}e\] — \[480.\overline{9}e\] — \[9.\overline{2}e\]

1. In the firste there appeareth, 2. numbers, that is \[14.\overline{2}e\].
of Co$ike nombers.

1 4.£ = 15.£, equalle to one nomber, whiche is 7 1.£. But if you marke them well, you maie see one denominatio, on bothe sides of the equation, which neuer ought to stand. Wherfore abating the letter, that is, 15.£ out of bothe the nombers, there will remain, 1 4.£ = = = 5 6.£, that is, by reduction, 1 £ = = = 4.£.

Scholar. I see, you abate, 15.£ from thei bothe. And then are thei equalle still, seyng thei wer equalle before. Accordeyng to the thirde common sentence, in the patthelwaic:

If you abate even portions, from thynges that bee equalle, the partes that remain shall be equall also.

Master. You doe well remeber, the firste groundes of this arte. For all springeth of those principles Geometricalle. Wherfore call to your minde likewaies the secunde common sentence, in the same booke, and then haue you another reason, whiche will helpe you not onely, in the other foymes of woork here, but also very often in the practise of this arte.

Scholar. That is this.

If you adde equalle portions, to thynges that bee equalle, what so amounteth of them shall be equalle.

Master. These twoo sentences doe instructe you that when you see on bothe the sides of the equation, as ny one denominatio Co$ike, you shall marke the signe that is annuered to the letter of them bothe: and if it be the signe of addition, =, then shall you abate that letter nomber, from bothe the partes of the equation.
As I did in this firste example. But if the signe be of abatemente, =, then shall you adde that letter no-bber, to bothe partes. And so shall you doe, till there bee noe one denomination on bothe partes, but diverse and distincere.

So the seconde nomber will be, 2 0.£ = = = 1 2 0.£ and in the leaste terme, 1.£ = = = .6.£.

Scholar. I see that you adde, 1 8.£, to bothe partes.
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But by that reason, I doubt in the thirde somme, because 10. is in bothe partes of the equation: in the firste parte with — — —, and in the seconde parte with — — , whether I shall adde 10. or abate them.

Master. In suche a case, you male doe either of bothe, at your libertie: and all will be to one ende.

Scholar. If I adde 10. then will it be 26. 9.


Master. And doe you not see 9. on bothe sides of the equation?

Scholar. I did loke but for one alteration only.

Master. If there were twentie like denominations, you should alter them all. For that is the principal and peculiare reduction, that belongeth to equations.

Scholar. Then must I abate 9. on bothe partes, and so will there remaine 17. 9. — — 20. 9.

— — 213. 9.

Master. How reduce it by abatynge 10. 9.

Scholar. So it will bee 17. 9. — — 213. 9.

— — 20. 9.

And now I remeber, that this is the better forme of reduction. Because the greater denomination, that is. 9. is alone with his number on the one side of the equation, and the 2. lesser denominations, on the other side.

Master. How doe you reduce the other equatios, to their smalleste sommes?

Scholar. In the fourth example, there is noe denomination, before the signe of equation, or in the firste parte, but the like is in the seconde parte also, after the signe of equation. Whereforst firste, because I see 19. 9. on bothe sides, I will abate it on bothe sides. And then will it be thus.


But
of Coike nombers.

But because I see 9, yet remaining on bothe partes, I abate the letter, that is. 10 9, from bothe partes, and it will be. 8 4 9. 10. 3 38. 2.

Mater. This equation would bee better, if the greater denomination, did stande as one parte of the equation alone. Whiche thyng you maie easly doe, by addyng. 38. 2. to bothe partes: because so mache foloweth— — — on the one parte.

And euermore when occasion serueth, to translate Translation nombers compound, on the one side is equallie of nombers. to —— on the other side.

Scholar. Then it will be thus.

8 4 9. — — 38. 2. — — 10. 3.

Mater. It were better thus.

10. 3. — — — 38. 2. — 8 4 9.

And in smaller termes.

5. 2. — — — 19. 2. — 42. 9.

But now procede with the examples.

Scholar. The fifthe is easly reduced, by abatyn 5. 2. on bothe sides: For so will it bee.


The sixthe equation will be, by addyng. 1 2. 2. on bothe sides. 3 4. 2. — — 5 2. 2. — 48 0. 9. — 9 2.

But yet I must reduce it farther, by addyng. 9. 2. on bothe sides. And then it will stande thus.

4 3. 2. — — 5 2. 2. — — — 48 0. 9.

Mater. Now will I shewe you the varieties of equatios, taucht by Scheubelius, because you maie perceiue, how they bee contained in those. 2. forme, naamed by me. As for the manysolde varieties, that some other doke teache, I accounte it but an idle bablyng, 92 (to speake moare favourobly of them) an unnestary

Varieties of

equations.
The first equation after Scheubelius, & after my meaning also, is, when one number is equal to another, meaning that they both must be simple numbers composed, and uncompounded. As, 6. \( \frac{1}{12} \) equal to 18\( \frac{1}{2} \):

4\( \frac{1}{2} \), 12\( \frac{1}{2} \), 14\( \frac{1}{7} \), 70\( \frac{1}{2} \):

15\( \frac{1}{2} \), 90\( \frac{1}{2} \):

20\( \frac{1}{2} \), 180\( \frac{1}{2} \):

26\( \frac{1}{2} \), 117\( \frac{1}{2} \).

In all these examples, as you see but one number, compared to another: so to finde the quantitie of one roote, you shall divide the number of the lesser Character, by the number of the greater Character, and so you shall the quotiente bying for the the quantitie of. 1. \( \frac{1}{12} \).

Scholar. It seemeth at the first seeme, that it is against reason to divide the number of the lesser signe, by the number of the greater. But when I consider, that if I compare a number of crownes, or any like denomination, to a number of shillings in equalitie, the number of crownes, or other soche like, must needes be lesser, then the number of shillings. And so dividing the number of the shillings (or other lesser name) by the number of crownes (or other greater name) the quotiente will shewe, how many shillings make a crowne: and generally, how many of the lesser, dooe make one of the greater.

As if. 20. crownes beo equalle to. 100. shillings, then. 5. shillings dooeth make a crowne. So when 6. \( \frac{1}{12} \), beo equalle to. 18\( \frac{1}{2} \), then. 3. \( \frac{1}{2} \), dooeth make. 1. \( \frac{1}{2} \).

And. 4\( \frac{1}{2} \), 12\( \frac{1}{2} \), dooeth cause that. 3. \( \frac{1}{2} \), must be a roote.

Master. As your cramplarie profe is good, so reduction will be a sufficiente profe in this.

Scholar. I se it manifeely. For if. 14. \( \frac{1}{2} \), be equalle to. 70\( \frac{1}{2} \), then. 1. \( \frac{1}{2} \), is equalle to. 5. \( \frac{1}{2} \), by that reduction
of Coßike nombers.

reduction in nombers. And againe by reduction in signes. 1. * is equalle to . 5. ⦿.

Likewaies. 15. * beynge equalle to. 9.0. * * re-
duction by signes and nombers also, will make 1. * 

--- . 6. ⦿. So shall 20. * * * * . 18. * . be reduced to. 1. * 

--- . 9. ⦿. And 26. * * . 10. 4. * * * . 

will make. 1. * . --- . 4. ⦿.

Walter. And so generally, when there is no de-
nomination omitted, betwene those. 2. that bee com-
pared in equalitie, still the division of the nomber, of
the lesser denomination, by the nomber of the greater
denomination, will byng so for the in the quotiente, the
quantitie of. 1. * .

But if there bee any denominations omitted, be-
twene those. 2. which be compared together in equa-
litie: loke how many denominations are omitted, and
so many in order is the rooted quantitie, whose roote
you must extract, for the auswerc were to the questio. For
in soche a case, euer more you shall extracte the roote
of your laste nomber.

As for example, when 6. * be equalle to. 24. * ,
by the former rule, you shall finde. 4. in the quotiente.
But here that. 4. is not the quantitie of a roote, but
is a rooted nomber, whose roote I shall extracte. And
byng betwene. * . and. * . there is no quantitie om-
mittend, but one, that is. ⦿. Therefore I shall ac-
counte. 4. the firste quantitie, that is to saie, a Square
nomber, and so take his Square roote, byng. 2. for
the quantitie of a roote.

Again if. 7. * . be equalle to. 567. * . the quotiente
will be. 81. and declareth a Zenzizenzike nomber, be-
cause there are omitted betwene. * . and. * , three
nombers: and Zenzizenzike is the thirde quantitie:
as you did leaerne in the beginning of this treatise, of
nombers denominate.

Scholar. I perceiue it. And therefore I must take
the
The Arte

the xenizenzike roote of 8, which e 3, and that is the true roote, where 7s, be equalle to 567.c.

Mater. And if those 7s were accepted equalle to 56, the quositie will be 8. And because there are omitted 2 quantities, that is, and., therefore you shall accopte that 8 to be 1d 02 a seconde quantitie. And his roote Cubike is 2, whiche standeth as the valewe of a roote, in the former equation.

And it is not possible that any other number, maie be placed as a roote, in that equation:02 in any other forme of this firste kinde. Hovwbeit in one sorte of equation, of the seconde kinde, there maie be 2 diverse rootes, when one number hath 2 rootes in valewe.

As I taught you befoxe, in the extraction of rootes.

The seconde kinde of equati, after Scheubelins minde and myne also, is, when one simple number Cubike, is compared as equalle to 2 other simple numbers Cubike, of severalle denominations, and like distance.

And in suche equation, being reduced as is taught befoxe, the roote of those 2 numbers compounded, as in one (or rather the valewe thereof) shall be extracted: As I have befoxe taughte also. And that roote dooth aunswere to the question.

Hovwbeit, here is the like observation, as was in the seconde forme of the firste kinde. For if those 3 denominations be not immediate, but doe omit some other betwene them, then shall you extracte the roote of that laste number, in all pointes, as you did in the firste equation.

Examples of the firste sorte.

4s. = - 6.5 - 4.9.

whiche being reduced, will bee:

1s. = 1/2.5 = 1.9. And the roote will be 2
And 6s. = 12.5 = 18.5.

That
of Cubike numbers.

That is by reduction.

\[
\begin{align*}
1.2.3. & = 2.3.2. + 3.3.2. \\
1.2 & = 2.2. + 3.3. & \text{And the route is .}
\end{align*}
\]

\[
\begin{align*}
5.2^3 & = 2.5.2. + 3.0.2. & \text{By reduction.} \\
1.5 & = 5.2. \\
1.3 & = 5.2. + 6.2. & \text{Whole route is .}
\end{align*}
\]

Likewise.

\[
\begin{align*}
2.2 & = 1.2.0.0. \\
2 & = 1.2.0. \\
1.2 & = 6.0.4. + 4.2. \text{Whole route is .6.}
\end{align*}
\]

Examples of the second route.

\[
\begin{align*}
5.2^3 & = 6.0.2 + 3.2.0.2. \\
\text{That maketh by reduction.} \\
1.2 & = 1.2.0.0. + 0.4.2. \\
\text{And the square route .} \\
8.2 & = 4.0.2 + 3.0.2.0.2. \\
\text{By the orderly reduction.} \\
1.2 & = 5.2 + 3.7.7.6.2. \text{Whole Cubike route is .}4.
\end{align*}
\]

Again in residuals.

\[
\begin{align*}
8.2 & = 8.6.4.2. + 2.4.2.2.2. \\
\text{That maketh by reduction.} \\
1.2 & = 1.0.8.2. + 3.2.2.2.6. \\
1.2 & = 1.0.8.9 + 3.2. \text{Whole route is .3.} \\
90.2 & = 9.0.2 + 1.4.4.2. \\
\text{By reduction.} \\
1.2 & = 1.0.2. + 1.6.2. \text{Whole route is .8.0.2.2.}
\end{align*}
\]

But now because Schubelius dooth make. 2. several equations of these 2. forms: And giveth. 3. diverse rules, 2. canons for each of them, I will declare his. 6. canons to be all contained in this second kind of equation.
The Arte

He maketh his division thus. When 1. number is compared as equal to 2. other, other that one number is of the smallesi denomination. And then is it of the firste Canon. As 1. 8. = 65. 9.02 els that one number, is of the greastes denomination: As. 3. 4. = 1. 1. 8. And then is it of the seconde Canon: Oi els thirdely, the alone number is of the middle denomination: and then is it of the third Canon. As 1. 1. 12. 8. 8.

The like some he thyleth, for the numbers of denomination is diuersyte.

Whereby you maye perceiue, that in my rule there is noe some of numbers, like the of the firste Canon, nother yet of the third: but onely of the seconde. But then again in my rule, there are Sortes of examples whiche he hath not. And if you compare them well to gether, you shall perceiue, that they bee agreable togethre.

As for example. In his firste canon, this is the some 1. 6. = 27. 9. whiche equation in my rule, by translation, is expressed thus.

1. 27. 9. = 6. because I doe still let the greateste denomination alone.

Again in his thirde Canon, this is an example.

1. 13. 9. = 8. and that number doe I translate into this some 1 8. = 15 9.

Now where as he giueth generall rules, for every Canon, I caste for them all: extracte the roote of that compoyse number. For all his rules doe teache nothing els.

Scholar. I doe understand the diversitie, and agremente of your rules and his. But for my exercise, I doe couet some apte questions, appertaining to these equations.

Master. Take this for the firste question.

Alexander byng aske how olde he was, I am 2. yeres
of Cossike nombers.

yeares elder (quod he) then Cephastio. Ver, said Cephastio. And my father was as olde as we bothe, and. 4.
yeares moare. And my father having all those yeares,
said Alexander, was 96 yeares of age. I demaunde
now of you, how olde was eche of them.

Scholar. I praie you answere the question your
self, to teache me the same.

Mater. I will begin with the ponest maner of
age, and that will I call. Where is the common 1 is the
supposition in all soche questions. Then is Alcran-
ders age. 2 yeares moare, that is 1 . Add 2. 9. And those
bothe together do make. 2 . Add 2. 9. Whereunto if you put. 4. more, then have you the age
of Cephastio his father, that will be. 2 . Add 6. 9. And all these put together, that is. 4. Add 8. 9.
will make 96 which is the equation that shall open
the question.

Wherefoze I set done the equation thus.
4. Add 8. 9. 96. 9. And because I see on
bothe sides, one denomination of. 9. I do abate. 8. 9.
from bothe sides: then there remaineth. 4 . 88 9.
And by reduction or division, 1 . 22. 9.

Scholar. Then make I easily cale, that Cephastio The proofe,
was. 2 yeares olde, byng you did putte. 1. Add for his
age: and now. 1 is founde to be. 2. Add therby all
the other yeares he manifeste. For Alexander byng. 2
yeares elder, must be. 2. 4. And Cephastio his fa-
ther had in age. 22. and. 2. 4: and. 4. more, that
is. 50 yeares. All which make. 96. So is that
question fully answered.

Mater. Another question is this. I had
a summe of money owinge unto me: whereof I did re-
ceive at one tym e 1 and afterward I received 2 of that
of deute. residue, which remained unpaid. And so remained
the reste of the deute 2 7. 1. I would knowe what was
the first deute, and what wer the. 2. General payments
S. if. Scholar
The Arte

Scholar. This muste I observe still, to name the first doubtfull thyng. 1.  \( \frac{1}{12} \). Wherefore I sate that the firste debte was \( \frac{1}{12} \). Whereof I received \( \frac{1}{2} \). And so did there remaine \( \frac{1}{2} \). Of whiche reste, againe I received \( \frac{1}{2} \), that is \( \frac{1}{2} \) of the whole somme, or \( \frac{1}{12} \). And that being abated also, then did there remaine \( \frac{2}{3} \). Whiche you named to be \( \frac{27}{1} \). Then if \( \frac{2}{3} \). Bsc. bee equalle to \( \frac{27}{1} \), diuide \( \frac{27}{1} \) by \( \frac{2}{3} \), and the quotiente will bee \( \frac{2}{2} \), that is \( \frac{3}{2} \). Whiche was the whole debte: And then is it plaine, that \( \frac{1}{12} \) of it is \( \frac{1}{5} \). And \( \frac{1}{2} \) of the residue is \( \frac{1}{8} \), whiche maketh \( \frac{3}{2} \). And then remaineth \( \frac{2}{2} \).

Waser. There is nothing better then exercise, in attaynyng any kynde of knowlidge: And therefore I will yowe you with diverse questions, to make you the moare experte in this rule. And this is one.

There is a floode Paved with Square Brickes, the lengthe of that floode being longer then the breadth, by \( \frac{1}{2} \), and the whole Pavemente containeth \( \frac{1}{2} \) 8 4. Brickes: I require to knowe the brede and lengthe.

Scholar. The letter quantyte, whiche is the brede I doe name. \( \frac{1}{12} \). And then the lengthe will bee, by your proportion. \( \frac{1}{12} \). \( \frac{1}{12} \). Now must I multiplie the brede by the lengthe (for that is the orderly worke in all flatte souymes, to finde out the whole platte) that is here. \( \frac{1}{2} \). \( \frac{1}{2} \). \( \frac{1}{2} \), and there will amounte the whole platte. \( \frac{1}{2} \). \( \frac{1}{2} \), whiche by your supposition is equalle to \( \frac{1}{2} \) 8 4.

Wherefore according to your rule, I diuide \( \frac{1}{2} \) 8 4. by \( \frac{1}{2} \), and the quotiente will bee \( \frac{1}{2} \) 3 6. Which is a Square number, because there is one denomination omitted in this equatio. For betwene \( \frac{1}{2} \) and \( \frac{1}{2} \), there is omit-ted \( \frac{1}{2} \). And therefore must I extracce the square root of \( \frac{1}{2} \) 3 6. And it will bee the quantyte of \( \frac{1}{2} \). That I knowke in my tables, and finde it \( \frac{1}{2} \) 6. whiche must be the brede: for that I named \( \frac{1}{2} \). Then the lengthe must be moare by \( \frac{1}{2} \) of it; and so shal it be. \( \frac{1}{2} \) 4.

Now
of Cosike numbers.

How so? to confrme my woorke, I multiplie.5 6. by.6 4 and it will make. 3 5 8 4. whiche is the nombre that you olost name.

Harker. That question is well answered: And if you had putt. i. 3. for the lengthe, as you might do, then the bzt the will be. 3. 3. and the square. 9. and so. i. 3. would bee. 6 4. as you maie proue at tetter: but in the meane time, what saie you to this question?

There is a capitaine, whiche hath a great armie, I would gladly Marshall the, into a square bataille, as large as mighte bee. Wherefore in his fisste profe of square-fonne, he had remaining. 2 8 4. to many. And prouyng again by puttyng. i. moare in the fronte, he founde wante of. 2 5. men. How many souldiers had he, as you seest?

Scholar. I call the fisste fronte. i. 3. and then multiplying it squarely: I shall haue for the whole bataille. i. 3. and so by your saying, there was left 2 8 4. men, whereas the whole nombre of men, was 1. 5. + 2 8 4. 3.

How so? the seconde profe, when the fronte was increased by. i. man: I shall sett the fozmer fronte, and 1. manne moare, that is 1. 3. + 1. 3. And multipliynge that nombre, 1. 3. + 1. 3. squarely: there will arise 1. 5. + 1. 3.

1. 3. + 2 3. + 1. 3. 1. 3. + 1. 3. out of whiche I muste a 1. 5. + 2. 3. + 1. 3. hale 2 5. that you saie, did wante, to make by that square bataille. And then it will be. 1. 5. + 2. 3. + 2 4. 3.

Now haue I one nombre of menne, expresset by. 2 Cosike numbers: Of necessitie therefore must these. 2. numbers be equall: sayng that renipente one armie. Therfore I set them thus.

Eg. 1 i. 1. 3. 

An other woorke of that question A question of an armie.
The Arte

\[ 1.\overline{5}. \overline{9} = 2.\overline{4}.\overline{9}. \]

And finding \(1.\overline{5}.\) on both parres of the equation, I doc abate it, and then standeth \(2.\overline{8}.\overline{4}.\overline{9} = 2.\overline{9}.\overline{4}.\overline{9}.

Yet again I see \(1.\overline{5}.\) on both sides of the equation, and therefore, seeing the letter of them hath the signe of subtraction, I doc adde \(2.4\) to both numbers, and then will there be \(3.\overline{0}.\overline{8} = 2.\overline{9}.\overline{4}\), that is \(1.\overline{5}.\overline{4} = 1.\overline{5}.\overline{4}\).

So that seeing \(1.\overline{5}.\) was set for the first front: the same front must be \(1.\overline{5}.\overline{4}\). whose Square is \(2.\overline{3}.\overline{7}.\overline{1}.\overline{6}\) unto which I muste adde the \(2.8.4\). that did abonde, and then will the whole number be \(2.\overline{4}.\overline{0}.\overline{0}.\overline{0}\).

For further trialle whereof, I take the seconde fronte to be \(1.\overline{5}.\overline{5}\), that is \(1.\overline{9}.\), more then the firste: and his Square will bee \(2.\overline{4}.\overline{0}.\overline{2}.\overline{5}\). And so is there \(2.\overline{5}.\), more then the inske number of the armie, as the question supposed.

Master. That question maye be wrought also by naming the seconde fronte. \(1.\overline{5}.\) and then will his Square bee \(1.\overline{5}.\) but seeinge there wanteth \(2.\overline{5}\). menne, to make that Square bataille, the number shall bee \(1.\overline{5}.\overline{5}\).

Then for the firste front, you must set \(1.\overline{9}.\) man leste, as the question importeth, \(\overline{5}\). that will be \(1.\overline{5}.\overline{4}\) whose square will be \(1.\overline{5}.\overline{9}\) \(1.\overline{9} = 1.\overline{9}\) \(\overline{2}.\overline{9}\).

\[ \begin{align*}
1.\overline{5}.\overline{9} & = 1.\overline{9} \\
1.\overline{5}.\overline{9} & = 1.\overline{9} \\
1.\overline{8} & = 1.\overline{2}.\overline{9} \\
1.\overline{8} & = 1.\overline{2}.\overline{9} \\
1.\overline{8} & = 1.\overline{9} \\
1.\overline{8} & = 1.\overline{9} \\
\end{align*} \]

Into whiche I must adde the \(2.\overline{8}.\overline{4}\.\) menne that did a-bounde, whè that bataille was framed, and then will the
of Cosike number.

the number be. 1.285. 285. 2.2.2. And it must bee equalle to. 1.2.2.2.2. 2.2.2.2. Wh. to re reduce that equation, strike I add to both the sides. 2.2. Then refresh. 1.2.2.2.2. 1.2.2.2.2.2. Then I add. 2.2. because I will have noe- in the equation: and it will be,

1.2.2.2.2. 1.2.2.2.2. 310.2.2. Thirdly I abate. 1.2.2.2.2. on both sides of the equation: and then remaineth. 2.2.2.2. 310.2.2. That is. 1.2.2.2.1.55. 2.2. whereby it appeareth that the second fronte was. 155 and the first fronte. 154. so forthe, as you brought it before.

Another question is this.

There is a king with a great armie. And his ad- uersarie corruppeth one of his heraultes with giftes, and maketh hym swere, that he will tell hym, how many Dukes, Erles and other soldiars there are in that armie. The heraulte lothe to lease those giftes, and as lothe to bee untrue to his Prince, disieth his auns were, which he was true, but yet not so plain, that the aduersarie could thereby understande that, which he desered. And that auns were was this.

Looke how many Dukes there are, and soe ech of them, there are twffe so many Erles. And under every Erle, there are fouer tymes so many soldiars, as there be Dukes in the field. And when the muster of the soldiars was taken, the. 200. parte of them, was 9. tymes so many as the number of the Dukes.

This is a true declarati of eche number, quod the Heraulte: and I haue discharged wy the. Now gette you how many of eche fozte there was.

Scholar. Although the question seem harde, I see many tymes, that diligence maketh harde thinges easie, and therefore I will attempte the worke of it.

And strike for the number of Dukes I sette. 1.2. then will the number of Erles bee. 2.2. that is. 1.2. by
The Arte

by. 1. \( x \) multiplied twise: And the number of soldiers are. 8. \( x \), that is, 2. \( x \), multiplied by. 1. \( x \), twise. ymes, but because the. 200. parte of the soldiers is. 9. ymes so moche as the number of the Dukes, therefore must the. 200. parte of. 8. \( x \) be equalle to. 9. \( x \). And so consequently. 8. \( x \) == 1800. \( x \) and 1. \( x \) == 225. \( x \) 0. 25. \( x \) == 225. 9.

For if I set \( \frac{1}{9} \) and. 9, as equalle together, \( x \) would by the arte of fractions, bynge the same proportion in whole numbers, I shall have for. 9. this fraction \( \frac{1}{9} \). And sying the denominator, is all one in \( \frac{\text{1}}{320} \) and in \( \frac{\text{1}}{320} \) the proportion consisteth betwene the numerators.

Then to procede, if. 225, be equalle to. 1. \( x \). I shall take the square root of. 225, so. 21. \( x \), and that is. 15 which must be the number of Dukes.

And so have I the firste number, whereof the seconde number, that is the number of Earles, must bee 15 tymes. 15, twise: that is. 450. And the number of soldiers shall be. 4, tymes. 15, multiplied by. 450. that is. 27000. And so put the trialle of this woorkke, I take the. 200. parte of the soldiers that is. 1350, and I finde it to bee. 9. tymes. 15. that is. 9, tymes so moche as the number of the Dukes. And so is that question solved, and tried.

Master. This is an other question. There is a gronde inclosed with. 4, wallles, byng like lambes and of one heigthe. The longest. 2, wallles are in proportion to the shorteste, as. 5, to. 3, And unto the height thei bee double Sesquialter. Now multiplying the longeste by the shorteste, and that totalle by the heigthe, there will rise. 3993 \( x \) foot. I demaunde then, what is the lengthe and the heigthe of ech wallle?

Scholar. The least quantity is the heigthe, whiche I call. 1. \( x \), and unto it the longeste walle is double Sesquialter.
of CoBike nombers.

Sesquialter: that is, 2 ½. Now that same longeste is in proportion Superbipartientae quintas, to the shorteste wall. So must the shorter wall be 1½. Then must I multiplye all those numbers together, that is 1 ½ by 1½. Whereof doeth come, 25. Then shall I multiplye that laste, by 1½. And it will be 39 930. Which must be equalle, by the woodyes of the question, to 39 930.

So by reducynge them to one denomination, 1 ½. Shall be equalle to 5/8 that is, 5. 15 972 0.9. and 5. Shall extract the Cubike roote out of 10 648. Whereof I shall extracte the Cubike roote out of 10 648. And that is the quantitie of 1 ½. of the heighte of the walle.

In my Tables I wooske that extraction of Cubike roote, and finde it to be 12 2. And therefore must the longeste walle bee double Sesquialter to it, that is 55. And the shorteste walle will be 33.

For proofe whereof I doe multiplye 22. with 55. The proofe, and it maketh 12 1 0. Whiche number I shall multiplye by 33. And it will be 39 930 according to the supposition of the question.

Master, You doe choose still the leaste number, to be equalle to 1 ½. As the casuall forme. Howbeit you maie put 1 ½. for the lengthe of any of the walles.

And if you sette it fo, the longeste walle, then the shorteste walle will be 1 ½. and the heighte 1 ½. and forme of all those numbers will make, by multiplication together, 1 ½. equalle to 39 930. And so will 6. be equalle to 99 825. 0.9. and 1. Shall 166 375. 9. Whereof the Cubike roote is 55. and aunswereth to the quantitie of 1 ½.

But if 1 ½. be set fo, the measure of the shorteste walle, then the longeste walle will be 1 ½. And the forme of heighte 1 ½. And so all numbers multiplied together will make 1 ½. Shall 39 930. So shall 10 1 0. be equalle to 35 937. 0. And 1 Shall 35 937. Whereof
of the Cubike roote is. 33, and is the value of. 1.£. in this position.

Scholar. This varietie of woodke, is not onely pleasaunte, but it maketh the reason of the woodke to appearre moare plainly. So that I could never be we- ric to heare soche questions.

Walter. Then will I propounde one 922. moare before we passe from this kind of equation. Where-man shall be somewhat like that last. And this it is.

A Brickeleiar had a pile of Bricks, whiche he sold by the yarde. The lengthe of it was 7 to the brede, that is Tripla sesquialters. And the heighte was five tymes so moche as the legthe. This pile the owner sold for 98 0. crounes. By suche rate that he had for every yarde so many Crounes, as the pile had yarde in brede. Now is the question, what was the lengthe, brede, and heighte of this pile?

Scholar. I suppose the brede to be 7. then was the lengthe 7 7. and the heighte. 17 7. These 3. sommes do I multiple togethe, and the make 24 7. whiche standeth as equale to all the yarde in the whole pile. But yet what that is, I knowe not.

Wherfore to procede farther, I consider that every yarde coste as many crounes, as the brede contained yarde. Now the brede being 7. I must saie, that every yarde did coste 7. of crounes. And then by the Golden rule: if 1 yarde coste 7. of Crounes, what shall 24 7. coste?

Woorke by the rule, I finde that it shall cost 392 0. And the question dooth suppose that it coste 98 0. crounes. Wherfore it must saie, that 98 0. crounes, are equale to 392 0. And consequently, 2 45 0. And wherfore dividenge the number of the lesser name, by the other, the quattuor will be 16. whose zenzizenzike roote is 2
of Cosike nombers.

And that therefore must be the value of a roote, and the bredthe of the pile. So shall the length be 7 yarde, and the heighte 35 yarde.

For trialle of it, I mutiplic the lengthe by the bredthe, and that totalle by the heighte, and so haue 490 for all the yarde of Wicke. Then considering that every yarde coste 2 crounes, because 2 yarde is the bredthe of the pile; the number of crounes must be twise 490, that is 980. And so is the woorkke good.

Mater. Now woorkke that question, by letynge 1. for the lengthe.

Scholar. If the lengthe be 1. the bredthe must bee \( \frac{1}{3} \), that is Subtriplae celualter to. 1. And the heighte must bee 5. All which somes make by multiplication \( \frac{1}{3} \).

Then farther, if 1 yarde coste \( \frac{1}{3} \), shall coste \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \), which is equall to 980. And so is 2. \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \), equal to 48020. And by division 1. \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \) = 2401. whose ceneizenike roote is 7. And that is the lengthe of the walle, and is the value of 1.

The reste of this woorkke, is like as before.

Mater. Yet proove the thirde walle.

Scholar. The heighte being 1. the lengthe must be the first part of it, that is 1. And the bredth \( \frac{1}{3} \). All these make by multiplication \( \frac{1}{3} \). Then for the price, if 1 yarde coste \( \frac{1}{3} \), what shal coste \( \frac{1}{3} \). By the Golden Rule there is founde, \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \), which is equall to 980. And so shall 4 \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \), be equall to 6002500. And 1 \( \frac{1}{3} \) \( \frac{1}{3} \) \( \frac{1}{3} \) = 150625. whose ceneizenike roote is 35. And that is the value of 1, and the heighte of the pile.

Mater. One question moare will I propounde,
A question of a Testament.

A poore man died, whiche had sower children, and all his goodes came to 72. crounes: whiche he would have parted so, that the seconde + thirde childe should have 7. times so moche as the first. And that the portions of the thirde and fourthe childe should bee 5. tymes so moche as the secondes parte; And that the first and the fourthe, should have twise as moche as the thirde. If you worke the solution wel, you make sorne worthy to be master of those warded.

Scholar. I trust to obtaine more benefite by the question, then by that office. Wherefore I will give good here unto it. And for the firste number, let 1. \( \times \) \( \times \) then muste the seconde and thirde portions make together. 7. \( \times \). And the fourthe must bee all the reste of the. 72. that is. 72\( \times \) 8 \( \times \). Now the thirde must be halfe the firste + the fourthe, that is. 36 \( \times \), 3 \( \times \). And the third + fourthe, is 5. tymes the second. Wherefore the seconde shall be the 5. part of 108 = 113 \( \times \). that is. 21\( \times \) = 2 \( \frac{1}{2} \) \( \times \). Whiche number I shall set in order with Letters, as here I have doen so, for my owne ease, and aide of memory. And then shall I add them all together. Whereof there commeth.

\[
\begin{array}{c}
| A | 1 \times 27 | 36 | 21 \times 27 | 129 \\
B | 21 \times 4 \times 27 | 36 \times 4 \times 27 | 129 \times 4 \times 27 | 129 \\
C | 36 \times 4 \times 27 |
\end{array}
\]

which is equall to 72. First therefor I do add all that followeth to bothe partes of the equation. And so haue I 129 \( \times \) 12 \( \times \), 27 \( \times \) 72. But because there are numbers Absolute on bothe sides, I shall abate the letter somme, that is. 72. from bothe partes, and then will there bee lefte, 7 \( \times \) 12 \( \times \), that is. 288 \( \times \) 64 \( \times \). And by division 4 \( \frac{1}{2} \) = 1 \( \times \).

The profe. So shall the firste mannes portion bee 4 \( \frac{1}{2} \). And the seconde and thirde mannes portion 7. times so moche that
of Cossike numbers.

that is. 31. Whereby it followeth, A 4.
that the fourthe manne, shall have 4. 11.
the reste of 72, that is. 31. 36. 31.

Then seyng the thirde manne, D 36.
hath halfe so moche as the first and 72.
the fourthe, his portion shall be 20.
And then by diverse reasons, the second mannes part shall bee. 12. And all these partes added together, do make make. 72. Wherefore the woork be is good.

Master. You have wroughte it well. And yet another
make you woork it thus. Firste lette doune. 1. se forse of
the first mannes parte. And then for the second and
thirde loynly. 7. 2. So shall the fourthe manne have
72. 8. 2. And because the second mannes
parte is 1 of the thirde and fourthe mannes portion,
if you loyn all their 3 partes together, the second
mannes portion will be 1 of that totall. But therefor
7. 2. which is the partes of the second and the third
byno. 72. 8. 2. which is the fourthe mannes
parte, and the totall will be. 72. 9. 1. 2. whose
sirte parte is 12. 9. 1. 2. for the second man-
nes share. Whiche somme if you abate out of. 7. 2.
there will remaine for the thirde

And so have you euer man-
nes portion allotted to hym due-
ly. As I have here set it for the
for you. And all the added to-
gether, do make. 72.

Scholar. But here is noe equatio yet, though the
partes be divied justly.

Master. Now shall you see it.
The question saith, that the thirde mannes portion
is halfe the portions of the first and fourthe man.
Wherefore seyng the firste and fourthe mannes portions do make. 72. 7. 2. the thirde mannes po-

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...tion being doubled, shall make as moche. But the double of the third maines parte, is 14½£ — 24£.

and therefore I saye, that those. 2. numbers be equall:

that is: 72.£ — 14½£ — 24£. Firste addde: 7.£ to every parte, and it will bee 72.£ — 21½£ — 24£. Then addde 24.£ on bothe sides, and there will be: 96.£ — 21½£. That is by reduction. 288.£ — 64.£ as you made it.

And then all agreeeth.

Likewaies for the equation, you make set the third 
mannes portion, with the halfe of the firste & fourthe 
mannes partes. And so will: 7½.£ — 12.£ be 
equall to: 36.£ — 3½£. And by reduction, 
10½£ — 48.£. That is in other terms of whole 
nomber: 32£ — 144. And by divicion it will bee 
1£ — 4. And thus will we ende the examples 
of the firste equation, for this tyme. And will shewe 
you some questions of the seconde equation.

Examples of the seconde equation,

by questions propounded.

A question 
of silkes.

Here are two men that have silke to sell. The one hath, 40. elnes, 
and the other. 90. And the firste 
man his silke is not so fine as the 
seconde man his silke. So that he 
seileth in every angell, price more 
by½ of an elne, then the seconde ma 
doeth. And at the ende, bothe their moneis made but 
42. angelles. Now I demaunde of you, how moche 
cehe man solde for an angell?

Scholar. I will folowe my olde forme, in putting 
1.£ for the leaste quantitie, where is the seconde 
mannes somme, and then shall the firste maines 
somme be, 1½£.

Master. You are deceived all ready. For you set 
1.£.
of Cosike nombers.

1. $\frac{1}{2}$, so an elce. Seyng you name $\frac{1}{2}$ of an elce, to be $\frac{1}{2}$. And so were the position headlesse, and likewe all the woork.

Scholar. I see mye faulte: but I knowe not how to amende it. For that, $1 \frac{1}{2}$, maie bee a parte of partes of an elce; and so maie it be moare then $1 \frac{1}{2}$. Elces so that I ought not to have set $\frac{1}{2}$ (which is certeinly referred, in this question, to an elce) as the parte of a doublull quantitie, but rather as the parte of a quantitie certaine. Whereas, $1 \frac{1}{2}$, is euery put for a numbeer unknowne.

Walter. To helpe you herein, I will set the firste numbers, as you began thew. The seconde man his numbers of elces, shal bee $1 \frac{1}{2}$, as you did name it, and then thall the firste man's portion be as moche, and $\frac{1}{2}$ of an elce moare, which $\frac{1}{2}$, maie best be called $\frac{1}{2}$. And so thall it bee distaunte from $1 \frac{1}{2}$, clerely in all woork Arithmetical.

But now to proceede, I shal divide eche mannes number of elces, which he had, by the number of elces, which he solde for an angelle, and the quotiente will declare how many angelles eche man had receiued. So that the firste mannes number of elces, beeing $40$, shal bee the numerato, and the somme of measure, which he solde for an Angelle, that is $1 \frac{1}{2}$, shal bee the denominato. And so is the division camed. And that fraction is the quotiente.

Scholar. Now I perceive the woork. And by like reason: the seconde mannes somme of elces beeing $3 \frac{1}{2}$, shal bee the numerato, and $1 \frac{1}{2}$, beeing the somme of measure, solde for one Angelle, shal bee the denominato, that is in one fraction $\frac{1}{2}$; accordingly as I have sette bothe nombers here.
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Master. It were more ease for you in working, if you did tournec that fraction, into an intere unitie.

Scholar. That will easily be done, by multiplying every number, of that whole fraction by. 3. And then will it be \( \frac{3x}{4} \), which is all one in value with \( \frac{x}{4} \). And this I consider farther, that as

\[ \frac{1}{2} + \frac{1}{3} \]

these 2. fractions, severally do express the sommes of angelles, that eche of them received, so jointly both together, doe declare the full somme, of all their angelles. Wherefore I shall add theim both together. And thei will make.

\[ \frac{390}{30} + \frac{90}{30} \]

As here in wooske I haue expressst.

\[ \frac{390}{30} = 13 \]

And by your supposition, theire both the sommes of Angelles made. 42. So that those 2. sommes are equalle: and therefore am I come to an equation. In whiche I see a number absolute, equalle to a fraction 

\[ \frac{390}{90} \]

And by your supposition, their bothe sommes of Angelles made. 42. So that those 2. sommes are equal: and therefore am I come to an equation. In whiche I see a number absolute, equalle to a fraction Coike compound.

Master. When so ever that, oz the like dooth chaunce, you shall reduce the whole nober, to the like denomination: and then their numerators will bee equalles.

Scholar. Then shall I multiplie, 42. by the denominator \( \frac{390}{90} \) it wil be \( \frac{126}{30} = 42 \)

Whiche muste bee equalle to \( \frac{390}{90} \).

That is in letter terms.

\[ \frac{21}{7} \]

Where firste I dose abate \( \frac{7}{4} \) on bothe sides: and there remaineth then \( \frac{21}{5} \).

But
of Cosike nombers.

But now I remembe your admonition, that because the number annexed to the greateste signe, is moare then. 1. I shall divide all the numbers by it, and sette their quotientes in their stede, with their signes. And so will the number of the greateste signe, euermore be 1. And this equation will be 1.8. --- 11.2. --- 11.9. Where I must extracte the square root of the latter part, according to your doctrine, and it will be 3. As it appereth in this worke following, whiche I did frame in my tables.

In square doeth make 3, onto whiche I must adde 3, whiche is all one with 3, by reduction to one denominatiou. So is the full additiou 3.3. whose square root is 3, unto whiche I shall adde 3, and it will bee 6, that is 3.

Mister. This is well done. How vsoke the same questio, as it was propone, and you shall easily finde all the other numbers to bee true, and agreable to the question.

Scholar. Seyng the seconde manne solde. 3. elnes The profe. for an angell, the firste manne did sell. 3. elnes and 1. So. 4.(whiche is the somme of elnes of the first man his silke) divided by. 2.9. doeth yelde. 12. and the weth how many angelles that man receiued.

Againe for the seconde man, whiche had. 9 o. elnes, diuide that. 9 o. by. 3. and so shall you finde. 3 o. for the number of his Angelles. And that. 3 o. and. 12. dooe make. 42. it neadeth not to be proued.

Mister. How againe for your exercise, suppose the firste mannes somme to be. 1.2e.

Scholar. Then multyke the seconde manne sell for an angelle. 1.2e --- 14.9. And their numbers of elnes, diuided by those numbers will make. 12. and 12. whiche bothe added together, will bee 3.90.2e --- 40.9. --- 126.2e. --- 42.2e. That is by reduction. And
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And by addition of $42 \frac{3}{2}$ on bothe partes,
$432 \frac{1}{2} - 409 = 126 \frac{1}{2}$. And by division it
will be $3 \frac{1}{2} - \frac{9}{3} \frac{1}{2} = 1 \frac{1}{2}$.

So that now I must ertrate the roote of that compounde Cobike fraction, thus, $\frac{11}{7}$ squarely, doce make
$\frac{11}{4}$ out of whiche I shall abate $\frac{1}{6}$. And therefore, first of
all I doe reduce the to one denomination, thet make
$\frac{11}{8}$ and $\frac{3}{4}$. Wherefore if I abate the lesser out of the
greater: there will remaine $\frac{2}{5}$, that is in lesser ter-
nes $\frac{11}{4}$ and is a square number, whose roote is $\frac{11}{4}$ un-
to whiche if I add $\frac{1}{7}$ that is $\frac{3}{4}$ it will make $\frac{1}{11}$, or $\frac{1}{1}$, that is the bale we of. $1 \frac{1}{2}$. And is the sifte mannes
number of elnes, agreably as I tried it before. And
so doe bothe workes agree.

But now commeth to my remembrance, that this
number, whose roote I did ertran, in this laste worke
is of that sorte, where $\frac{3}{2} - \frac{1}{9}$ is equalle to $\frac{3}{2}$.
And therefore hath in it $2$. rootes: thone by addition,
as this, whiche I now founde: And the other by sub-
traction, whiche in this example by abatyng $\frac{1}{7}$ out of
$\frac{3}{2}$, will bee $\frac{1}{7}$. But how I make frame that roote, to a-
gree to this question, I doe not see.

Daster. That varietie of rootes dooth declare,
that one equation in number, maie serve for 2. seve-
ralle questions. But the forme of the question, maie
easily instruct you, whiche of those 2. rootes, you shall
take for your purpose. Howbeit sometimes you shall
take bothe. As for example again, marke this ques-
tion.

A gentilman, willyng to proove the cunning, of a
braugynge Arithmetician, saide thus: I haue in bothe
my handes, 8. crownes: But and if I accounte the
somme of eche hande by it selfe seuerally, and put ther-
to the squares and the Cubes of the bothe, it will make
in number. 194. How tell me (quod he) what is in
eche hande; and I will give you all for your labour.

Scholar.
of Cosike nombers.

Scholar. Soche incoragementes, would make me stude harde, and travell very willingly in learned exercises: though learning bee moste to be loved, for knowledges sake. But so to finde the true aunswere thus I doe proceade.

Firste I suppose the one nomber in one hand, to be \(1 \cdot x\). And then must the other neses be \(8 \cdot y = 1 \cdot x\). Then doe I make them bothe Squares. And for the firste I haue \(1 \cdot x\). And for the seconde \(1 \cdot x = 64 \cdot y\) -- \(16 \cdot x\). Thirdely I multiply them bothe Cubes: and so haue I for the firste \(1 \cdot x\) and for the other \(24 \cdot x\) -- \(512 \cdot y\) -- \(1 \cdot x\) -- \(192 \cdot x\).

Then must I adde bothe the nöbers, with their squares, and their Cubes, into one somme. As here in work

\[
\begin{array}{c|c|c|c|c|c|c}
1 \cdot x & | & 1 \cdot x & | & 1 \cdot x & | & 1 \cdot x \\
8 \cdot y & | & 1 \cdot x & | & 8 \cdot y & | & 1 \cdot y \\
1 \cdot x & | & 64 \cdot y & | & 16 \cdot x & | & 16 \cdot y \\
24 \cdot x & | & 512 \cdot y & | & 1 \cdot x & | & 192 \cdot x \\
26 \cdot x & | & 584 \cdot y & | & 208 \cdot x \\
\end{array}
\]

It is set for the. Where for case I haue set \(1 \cdot x\), \(1 \cdot y\), and \(1 \cdot x\) (which is the Roote, the Square and the Cube of one nomber) all in one line: and the other Roote, Square, and Cube, I haue set severally. And so all thei doe make \(26 \cdot x + 584 \cdot y = 208 \cdot x\) which is equalle to \(194 \cdot y\) by the intente of the queston. Wherefore I adde firste \(208 \cdot x\) to bothe partes, and there remaineth.

\[
26 \cdot x + 584 \cdot y = 208 \cdot x + 194 \cdot y.
\]

Then I abate \(194 \cdot y\) from bothe sides, and so reflete the equatio thus: \(26 \cdot x + 390 \cdot y = 208 \cdot x\) That is by divition \(1 \cdot x = 15 \cdot y\) \(= 8 \cdot x\). And by translation of \(15 \cdot y\) tosette \(1 \cdot x\) alone, it will be \(1 \cdot x = 8 \cdot x\) -- \(15 \cdot y\). And now haue I the crate and complete equation, where I must seke fo2

\[x, y\] the
The Arte

the value of \(1.2\) by extraying the roote. Therefore firste I take halfe of \(8\) and that is \(4\), whose square is \(16\). out of whiche I abate \(15\), and the remainder is \(1\), whiche I maie either adde to \(4\) and so haue \(5\). other, I maie abate it from \(4\) and so haue \(3\). Whiche numbers also according to the same rule, byng added together doo make \(8\). that is the number of the mid-dell denomination. And byng multiplied together, thei doo make \(15\). that is the other parte of the same compounde Coiike number.

Waster. And if you had marked that firste, you might easilly haue found bothe those numbers, by the partes of \(15\). whiche can be none other, but \(5\). and \(3\).

And farther, byng thei \(2\). doe make \(8\). And \(8\). is the number (named in the questio) that thei should make, thence you shall take them bothe. And name whiche of thei you like to be \(1.2\). And the other shall be of necessite, the reste of \(8\).

The profe. Scholar. To examine theim, by the order of the question, I doe proceede thus. \(3\). with his Square \(9\). and his Cube \(27\). dooeth make \(39\). And \(5\). with his Square \(25\) and his Cube \(125\). doe yeilde \(155\). And bothe thei together doe byng sothe \(194\). according to the saying of the question: and thence it is certain, that the wooke is good.

Waster. Before you passe any farther, I will ad-moniste you of one waile, whiche I ofte use in reduc- tion of soche equations, as this is, when there is noe denomination on the one side, but the like is on the o- ther side, with a greater number annered to it. Then mate you abate all the letter nobers, out of their grea- ters, and the reste shall bee accounted equalle to no- thing. Whiche chance can never happen: excepte there bee some nombers on the greater side, with the ligne of abatmente.

As here you had.
of CoFike nombers.

26 5  =  584 9  =  208 2  =  194 9.

Because on the one side, there is no number but 1949 and on the other side, there is 5849, being a greater number, and of the same denomination: therefore make you abate 194, from both sides, and then remaineth. 26 5  =  39 2  =  208 3  =  0.

Therefore you may well consider, that the numbers which be joined with ——— are equal to the numbers that be set with ———. And therefore the one abatting the other justly, dooe remaine together as equal to nothing.

Wherefore it is reasonable, that serving the numbers with ——— bee equalle to the numbers with ——— that I maye translate the numbers with ——— from that side of the equation, and set them on the contrary side, with the signe of ———. And so in this example it will bee. 26 5  =  39 2  =  208 3.

And this fomne shall ease you moche, in reducynge of equations.

Scholar. I thanke you moche. And I will not sowe to bie it, as occasio shall happen. But I prate you propounde yet some moare questions, that I maye see their diverse varieties.

Master. There were twoo severalle men, which had certaine sommes of angelles, in soche rate, that the seconde manne his somme, was triple sesquiquarta to the firste: and if their 2 sommes were multiplied together, and to that total the 2 firste sommes added, there would amounte .142 1. I demaunde of you, what was eche of their sommes in angelles?

Scholar. The firste mannes somme I call. 1 3. And the seconde mannes some shall be. 3 1 3. Which 2 sommes being multiplied together, doe make 3 3 3, unto whiche I must adde both the firste numbers, that is 4 1 3. And it will be 3 1 7 4 3, equalle to .142 1. All whiche numbers, I that binging
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into whole numbers, if I multiply them by 4. And so will it be, $13_2 = 57_0$. And by reducyng the greater denomination (osike, to an binitie). $1_2 = 43_3$. And laste of all, by translating the number of $z_2$, to set $1_z$ alone, on one side of the equation, it will be $1_z = 43_3 + rac{17}{z}$. Where I must extract the value of the roote thus, $17_2$. Squely dooe make $25_2$, unto whiche I shall adde the $43_3$, (it beeyng firste multiplied by 5 2, to byng it to the denomination of 676. And so makynge $\frac{10640}{676}$), And it will be $\frac{10640}{676}$, which is a square number, (as I haue poued in my Tables) and his roote is $\frac{17}{z}$, from whiche roote I must abate $\frac{17}{17}$, and then will there remain $\frac{156}{20}$, that is 6.

And that 6. is the value of $1_z$, and standeth for the firste mannes number. So that the seconde mannes number must be as $\frac{1}{4}$ to it; that is tripla sesquiquarta.

And so shall it be, $19_2$.

The profe. Master. How poude those numbers, according to the question.

Scholar. $19_2$, multiplied by 6, dooth make $117_2$, unto whiche I shall adde the $25_2$, amounting of theire 2 additioes, and all will be $142_2$, according to the purpose of the question.

Waster. So is your woorke good. Yet woorke it werke of the same question. Scholar. I maie put $1_z$, to betoken the seconde mannes somme. And then shall the firste mannes somme bee $\frac{1}{4}z$. Whiche bothe multiplied together doe make $\frac{1}{13}z$. And then addyng the 2 firste sommes to it, it will bee $\frac{15}{13}z = 1\frac{4}{z}$. And that is equalle to $142_2$. All whiche numbers will bee reduced to whole numbers, by multiplication conveniente. And so will it be, $8_2 = 34_2$, equale to $3705$; that is by reduction, $1_2 = 46\frac{4}{9}$, and by translation of the terms.

$1_z$. 
of Cossike nombers.

1₂₃₄ = 46.89. 4½₂₉, out of whiche number I shall extrack the value of the roote, in this sorte.

Firste I saie I multiplied Square, doeth make 4₂, unto whiche number I must abde 46 ¾, reduced as it ought, and it will bee in all 2². Which is a square number, and hath for his roote 4, from whiche I must abate ⅟. And then will there remain 2², that is 19 ⅞, for the value of 1₂₉. And so consequently for the second mannes nöber, which was named in this position, 1₂₉. And this made bee proued as the other was.

Master. What late you then to this question?

There is a strange tojneye appointed to a manne. The firste date he must goe 1 ½ mile, and every date after the firste, he must still augemente his tojney, by ½ of a mile. So that his tojney shall proceede by an Arithmeticalle progression. And he hath to travell for his whole tojney, 29 5/₂ miles. I demand: in what nöber of dates, shall he cande his tojney?

Scholar. I knowe not how to procede in this question.

Master. Doe you not heare me name it, ar Arithmeticalle progression: Whereby you might be assured, that it doeth appertaine to that rule. And accoording to the canons of that rule, must you wooke this question. But for your better instruction, I will helpe you in this wooke.

Firste answere to the question, by the common position: and saie that the tyne of his tojney is 1₂₉ of dates. And then shall all the excesses (whiche may also be called the number of the spaces) be 1₁₂ₙ. The common excess was supposid to bee ½ of a mile. And therefore the somme of all the excesses muste bee ⅛ that is to saie, the number of all the excesses multiplied by ⅘, that is here, the sixte parte of the number
number of the excesses.

And because that the firste number is \( \frac{1}{2} \), I must add it unto the summe of the excesses, and so haue I the laste number of that progression. Wherefore addyng. \( \frac{1}{2} \) (which is \( \frac{1}{2} \) of the like denomination with the other, \( \frac{1}{6} \)) with \( \frac{1}{2} \) it will make \( \frac{1}{6} - \frac{1}{6} = \frac{1}{6} \). And that is the laste number of the progression.

Now you remember, that in progression Arithmetical, if you add the firste number to the laste, and multiplie that totalle, by the number of halfe the places, there doeth amounde the summe totalle of that progression.

And therfore in this example, if you add \( \frac{1}{2} \) (which is the firste nober in the progression) unto \( \frac{1}{6} \) (that is the laste number of the progression) there will amounde \( \frac{1}{6} \), which beynge multiplie by the number of halfe the places, that is \( \frac{1}{2} \). (For all the number of places is \( \frac{1}{2} \)) there will rise, \( \frac{1}{6} \), which is the totalle somme of all the miles: and therfore is equall to 2955.

Scholar. All the reste, and this againe can I doe now. Seyng \( \frac{1}{12} \) is equall to 2955. I will firste beynge the whole nombere to the like denomination, with the fraction, and it will bec. \( \frac{1}{12} = \frac{1}{12} \).

And then omitting the like denomination. \( \frac{1}{2} \) - \( \frac{1}{12} \) = \( \frac{1}{2} \). That is, by translation \( \frac{1}{2} = \frac{1}{2} \). That is, 17 \( \frac{1}{2} \).

Whose roote in value I shall finde out thus. I multiplie \( \frac{1}{2} \) squarely, and it will be \( \frac{1}{4} \), unto which I shall addde 35 46 0. It it will make \( \frac{1}{2} \), which is a square number, and hath for his roote \( \frac{1}{2} \), fro which I shall abate \( \frac{1}{2} \), and then remayneth \( \frac{1}{2} \), that is. 18 0. Which is the value of \( \frac{1}{3} \).

And expresseth the number of vapes, which the question requirith.

The profe. Master. The profe in this, and the like questions, is, to setfoothe the progression with all his terms.
of Coßike nombers.

mes. Excepte you will for shorthead, lette doune the firste terme, whiche in this example is \(1 \frac{1}{2}\): and then by the number of the excesses, or distances (whiche is ever one lesse then the number of places) multiply the quantitie of one excess : and put to it the firste terme: and so have you the laste terme. Then haung the firste terme and the laste, with the number of excesses you knowe how to finde the totalle.

As in this example, the number of excesses being 179. And the quantitie of one excess being \(\frac{1}{2}\), their multiplication giveth \(\frac{179}{2}\) unto whiche if you adde the firste number, that is \(1\frac{1}{2}\), it will be \(180\). And that is the laste number of that progression. Then to trie the to-talle somme of the miles, adde the firste number, \(1\frac{1}{2}\) to the laste, and thei will make \(1\frac{1}{2}\), that you shall multiplie by halfe the number of the places, whiche in our example are 90 (thothe whole number is 180) and there will amounte 2955. accordyng as the question saith.

Scholar. This is sufficient for this question. And at some idle time, I will not sticke to trie it out, by set-tyng the progression soothe at large. In the mean time I praye you for better exercise, give me some moare questions.

Maste. There is a number, whiche I haue for- gotten: and it is divided into 2. partes, whereof the one I haue forgotten also, but the other was 4. And yet this I remember, that if the parte, whiche I haue forgotten, be multiplied by itself, and then also with 4. those 2. sommes will make 117. How would I knowe what was the whole nomber, and also what is the parte, whiche I haue forgotten.

Scholar. I suppose the whole nomber to be \(1\frac{1}{2}\). And because 4. is his one parte, the other parte must heades bee \(1\frac{1}{2}\). — 4. Then doe I accordyng to the question, multiplie \(1\frac{1}{2}\). — 4. firste by it self, lik.) and
The Arte

and it will make. 1: 3 = 16. 9 = 8. 3e. Secondly, I doe multiply it, (that is. 1. 3 = 4). by 4
And it giueth. 4. 3e = 16.

Then add I bothe those nombers together, and it will be. 1. 3 = 4. 3e. Whiche by the question shal
be equalle to. 117.

1. 3 = 16. 9 = 8. 3e.
4. 3e = 16. 9.

1. 3 = 4. 3e.

But then must I use the accustomed tranlation,
to bring the greateste quantitie in denomination, to
stande alone. And so will it bee.

Where I must resche for the value of a roote. And
therefore I multiplye. 2. by it selfe squarely, and so have
1. 4. unto which I addde. 117. and it maketh. 121.
Whose roote is. 11. unto which I must addde. 2. and
there commeth. 13. as the value of. 1. 3e. and the quan-
titie of the whole nomber.

The profe. For profe of this worke, I abate. 4. out of. 12. and
there resteth. 9. as the other parte. Then doe I multiplie.
9. by it selfe. and therof resteth. 81. Also I doe mul-
tiplie. 9. by. 4. and it maketh. 36. whiche bothe toge-
ther, doe make. 117. as the question would.

Master. Se. 1. 3e. for the unknowne parte. and
then woonke it, to see the diuerstie of the wooke.

Scholar. If. 4. bee one parte, and. 1. 3e. the other
parte, then will the whole nomber be. 1. 3 = 4. 3e.
Wherfoere firste I multiplye. 1. 3. by it selfe, and it
yeldeth. 1. 3. Then doe I multiplye. 1. 3. by. 4. and
it giueth. 4. 3e. whiche bothe sommes together, doe
make. 1. 3 = 4. 3e. which is equalle to. 117.
And by translation. 1. 3 = 117. 3 = 4. 3e.
Wherfoere I doe multiplye 2. squarely. and it gi-

An other wooke.
of Cojsike numbers.

neth. 1, whiche added to. 117, maketh. 121, and the roote of that is. 11. from whiche I shall abate. 2. and there will reste. 9. as the other parte of the number. This is herie plain, & the prose of it as it was before.

Master. Then answere to this question.

There are 3 nobers in proportion Geometrical. And A question one of the extremes is. 20 ½. the other extreme, with thedouble of the middell terme, dooth make 22. Now would I knowe of you, what those 2. numbers bee.

Scholar. For trialle, I name the other extreme, 1. 2. And because it, with the double of the middle terme dooth make. 22. the middell terme shall bee 11. _ ½ 2. 12. 2. for his double is. 22. _ 1 2. whiche with. 1. 2. dooth make. 22.

Then to procede, I knowe the property of those numbers in proportion Geometrical to bee soche, that the multiplication of bothe the extremes, is equale to the square of the middell terme, wherefore I multiply the 2. extremes together, and there will risse. ½ 2. Then dooe I multiply. 11 _ ½ 2. by it self in Square, and it will bee. 121 9. _ ½ 2. _ 121 2. which musst bee equale to. 1 ½ 2. 0. 2. 0. ½ 2. Then to reduce it, I addde. 11. 2. on bothes sides, and it will be. 31 ½ 2. _ ½ 2. _ 121 9. and by translation. ½ 2. _ 31 ½ 2. _ 121 9. That is 1. 2. _ 125 2. _ 484 9.

Now resteth nothing, but to searche the value of 1. 2. Wherefore I take. 1 2. and multiply it Square, and so haue. 110 9. from whiche I must abate. 484. that is. 121 9. And there will remain 110 9. whose roote is. 1 2. which I shall abate from. 1 2. and there will remain. 1 2. that is. 4. for the other extreme.

Then for the middell terme, thus shall I doo. Mul. The proose.

neth. 4. and. 20 ½ 4. together, and there will rise. 81, whose roote is. 9. and is the middell nober. That 9 doubled will make. 18. and 4. joined thereto, giveth. 22

Ex. 4. So
The Arte

So are those 3 terms in progression Geometricall, according to the conditions limited in the question.

Master. Prove the worke now, how it will frame if. 1. 2. be set for the middell number. For it were fol- lie, to cry whether this question would admit of the 2. laste numbers. Although the rule doe declare that in suche sorte of equations, there is double valuation to eche roote.

Scholar. Yet I beseech you, let me examine it a little, to see the cause, why I must not adde them, and so take the roote.

Master. I must here with you so moche. By addition you see, there will rise \( \frac{2}{3} \), that is 1 2 1. And then the middell number will be, 4 9 \( \frac{1}{2} \). And so the proposition is \( \frac{2}{3} \), that is Dupla superquadripartiens nonas. Where as in the other 3. numbers, 4 9 2 0 \( \frac{1}{4} \), the proposition is Dupla sesquiquarta.

But in the question is one condition, that secludeth the roote, that riseth by addition. For the double of the middell terme, with the other unknowen extreme, should make 2 2 As in. 4. and 9. it doeth. But in 49 \( \frac{1}{2} \) and 1 2 1, it would be 2 2 0, that is 1 0. tymes so moche.

Scholar. And if you had saied in the question, that the double of the middell number, with the other extreme, would have made 2 2 0, then I should have taken this latter roote by addition, and not the first roote by subtraction.

And so I perceiue the varietie of conditions in the question dooth limete, whiche of the 2. rootes I shall of necessity take, and leave the other.

But now to variate that worke, I will set. 1. 2. for the middell terme. And then the double of it, with the other terme, will make 2 2. The double of 1. 2. is 2. 2. So must the other terme be 2 2 \( \frac{1}{2} \) — 2 2.

Then to seke out an equation, I multiply the 2. extremes together, that is 2 2 \( \frac{1}{2} \) — 2 2 \( \frac{1}{2} \) by 2 0 \( \frac{1}{4} \).

And
of Cossike numbers.

And there riseth \(4 \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{1}{8} \). And the square of \(1 \frac{1}{2} \), being the middell terme, is some perceined to be \(1 \frac{1}{3} \). And so the firste equation is,

\[ \frac{1}{3} = \frac{4 \frac{4}{5} \cdot \frac{1}{9}}{4 \frac{1}{8}}. \]

Wherefore I take halfe \(4 \frac{1}{8} \), that is \(\frac{1}{4} \), whose square is \(\frac{1}{8} \). And into it I putte \(4 \frac{4}{5} \cdot \frac{1}{9} \), whereby there commeth \(\frac{1}{8} \), whose roote is \(\frac{1}{4} \), from which roote I must abate \(\frac{1}{4} \), and so remaineth \(\frac{1}{8} \) that is \(9 \). As the value of \(1 \frac{1}{2} \). And for the middle number.

Then for the proofe: if \(9 \) bee the middell number, The proofe. the square of it, which is \(81 \), shall bee equalle to the multiplications of the extremes. Wherefore if I divide \(81 \), by \(2 \cdot \frac{1}{8} \), the quotiente being \(4 \), declareth the other extreme.

Master. You see some experte enough in this somme of worke. Therefore I will procede to other questiones, that differ some what from these.

There are 2. menne talking together of their monies, and neither of them willing to express his somme, but in this softe. The number of angels in my purse, saith the firste manne, may bee parted into soche 2. numbers, which being multiplied together, will make \(24 \). And their Cubes being added together, will make \(288 \). Then, quod the other man. And the like maie I saie of my money, saue that the Cubes of the 2. partes, will make \(539 \). Now I desire to knowe, what monie eche of them had.

Scholar. The firste mannes somme, I set to be \(1 \frac{1}{2} \), which I must parte into twoo soche partes, that the both multiplied together, maie make \(24 \).

Master. You erre verie moche. For it is not possible, that the partes of any Cossike number multiplied together, can make an absolute number. Wherefore in soche cases, where you perceiue that there is required, after the firste position, any multiplication to make an absolute number, you shall call the firste no.
The Arte

bers, by some other name of pleasure. As here you
male call the firste mannes somme. And the second
mannes somme. When they in their partition, be the
name of 12.

And as there are two questions in one, so shall you
make severall woorkes for them.

Scholar. Then shall I say, that the firste mannes
somme is 12, and it is doubled as he declared. Where-
fore for one number of that division, I set 12. And
then the other shall be 1/2, for as the one number mul-
tiplied by the other, both make 24. So 24 divided
by the one of them, will proceed by yng for the the
other.

Master. That is well remembered of you. For as
4, and 5, by multiplication, doth make 20. So 20 di-
vided by 5 bringeth to the 4, and divided by 4, it yeld-
deth 5.

Scholar. So 1/2 is but 4, and 1/2 is 5.

Master. Go forth then with the rest of the woorkes.

Scholar. The Cube of 12 is 12, and the cube
of 12 is 12, which 2, numbers I male not add to-
gether, but ill I have reduced them unto one deno-
mination: which then I shall doe, by setting 12 as a frauction thus 1/2. And then working after the
rate of fractions, in the firste reduction they will stande
thus. 12 + 12 = 12/2. And by farther addition thus.

And therto the woorke of bothe these 2, menues
sommes, are indifferent and agreynge. So that this
one woorke serveth for them bothe. But now they
will differ. For in the firste mannes woorkes, and so
in the woorkes for him 12 = 12 is equale to 288;
but in the seconde mannes woorkes, it must be accom-
peted equale to 539.

But firste to goe foward with the firste man. Se-
yng 12 + 12 is equale to 288. Therefore by
reduction
of Cosike numbers.

reduction to one denomination, \( \frac{\sqrt[3]{15}}{\sqrt[3]{2}} \) is e-
qualle to \( \frac{\sqrt[3]{6}}{\sqrt[3]{2}} \). And removing the common denomi-
nator, the numerators shall keep the same proportion: and therefore, \( \frac{\sqrt[3]{5}}{\sqrt[3]{2}} + \frac{280}{\sqrt[3]{2}} \) shall be equalle
to \( \frac{13824}{\sqrt[3]{2}} \). And by translation, to have the great-est
denomination alone, \( \frac{280}{\sqrt[3]{2}} = \frac{13824}{\sqrt[3]{2}} \) Where I shall take the value of \( \frac{1}{\sqrt[3]{2}} \), which shall
not be here accounted the square roote, but the zen-
Zicubike roote, or, the Cubike roote of the square roote,
according to the greateste denomination.

Wherefore, \( 140 \) in square, maketh \( 19600 \) from
which I must abate \( 13824 \). And there dooth remain
\( 576 \) whole square roote is \( 76 \), which being added
unto \( 140 \), doeth giue \( 216 \), and being abated from
it, it leaveth \( 64 \), of which bothe I must extrance the
Cubike roote, because in the equation there are 2 qua-
tities omitted. So that of \( 216 \), the Cubike roote is \( 6 \).
And of \( 64 \), the Cubike roote is \( 4 \). Where I see bothe
rootes serve so my purpose, that I shall take the both.

Master. And good reason. For as in letting \( 1 \) for
your position, you could not tell whether it were
the greater parte, or the lesser, so maie you not now
apply it to either of theim bothe, but take bothe roo-
tes for the 2 partes of your number.

Scholar. So doeth the firste mannes number ap-
pear to be. \( 1 \), semyng the partes bee. \( 4 \) and. \( 6 \), whiche
I maie examine thus. That thet, make. \( 24 \) by mul-
tiplication, it is easily seen. And that their Cubes added
together, doe make. \( 280 \) is sone perceiued: semyng the
Cube of. \( 4 \) is. \( 64 \); and the Cube of. \( 6 \) is. \( 216 \), whiche 2
numbers by addition, doe make. \( 280 \).

Master. Now prove the seconde mannes worke. The worke

Scholar. In his woode \( \frac{\sqrt[3]{5}}{\sqrt[3]{2}} \) is eualle to \( 539 \). And by reduction to one denomination, it is
eualle to \( \frac{13824}{\sqrt[3]{2}} \). So that. \( \frac{\sqrt[3]{5}}{\sqrt[3]{2}} = \frac{13824}{\sqrt[3]{2}} \). Is
eualle to. \( 539 \). And by translation.

\( 1 \frac{\sqrt[3]{5}}{\sqrt[3]{2}} \).
The Arte

1. \( \frac{3}{8} \) \( \times \) \( \frac{11}{12} \) = \( \frac{33}{12} \) = \( \frac{11}{4} \)

whole zenzicubike roote I seke, thus: \( \frac{33}{4} \) doth make in square \( \frac{11}{4} \)
from whiche I must abate \( \frac{33}{12} \), and then remaineth \( \frac{11}{4} \), whole roote is \( \frac{11}{4} \) into which I maie addde \( \frac{1}{4} \), and then will it bee \( \frac{14}{4} \), that is, \( \frac{7}{2} \). Whole Cubike roote is \( \frac{7}{2} \). And is one parte of the seconde manner number. And for the other parte, I shall abate \( \frac{3}{2} \) out of \( \frac{11}{2} \), and there remaineth \( \frac{7}{2} \). That is, \( \frac{7}{2} \). Whole Cubike roote is \( \frac{7}{2} \). And is the other parte of the seconde manner number. As it maie some be tried thus. For, \( \frac{3}{2} \) tymes \( \frac{7}{2} \) maketh \( \frac{21}{4} \). and \( \frac{7}{2} \), which is the Cub of \( \frac{7}{2} \)
added with \( \frac{7}{2} \), which is the Cub of \( \frac{7}{2} \), dooreth make \( \frac{33}{4} \), as the question intendeth.

The prooste.

A question of an armie.

Master. One other question I will propounde, of 2 armie beyng bothe square, and of like number. And is you abate \( \frac{1}{2} \) from the one armie, and addde \( \frac{1}{2} \) to the other armie, and then multiply them bothe together, there will amounte \( \frac{9853272}{2} \). I demaund of you, what is the fronte of those square battailes.

Scholar. I call the fronte \( \frac{1}{2} \). And then must the battaille bee \( \frac{1}{2} \). Now abatyng \( \frac{1}{2} \) from the one, it will bee \( \frac{1}{2} \). Then addyng \( \frac{1}{2} \) to the other, it will make \( \frac{1}{2} \). And if you multiply those numbers together, there will amounte by it \( \frac{1}{2} \). \( \frac{1}{2} \) -- \( \frac{3}{2} \) -- \( \frac{4}{2} \). Which somme must be equalle to \( \frac{9853272}{2} \).

\[
\begin{align*}
1. \frac{3}{8} \times \frac{11}{12} &= \frac{33}{12} = \frac{11}{4} \\
1. \frac{3}{8} &= \frac{4}{3} \\
\hline
1. \frac{3}{8} - \frac{1}{12} &= \frac{10}{12} - \frac{4}{3} = \frac{4}{3} = \frac{4}{3} \\
- \frac{1}{12} - \frac{4}{3} &= \frac{40}{9} \\
\hline
1. \frac{3}{8} - \frac{6}{3} &= \frac{40}{9} \\
\hline
And if you addde. \( \frac{40}{9} \) to bothe partes of the equation, it will be \( \frac{1}{3} \). \( \frac{3}{2} \). \( \frac{1}{6} \). \( \frac{40}{9} \). equalle to \( \frac{9853312}{2} \).
\end{align*}
\]
of Cosike nombers.

And by translatioun. 1 2 3 4 5 6.

out of whiche laste equation, I shall searche for the value of 1 2 3 4 5 6.

by multiplyng first 3, squarely, wherof commeth 9, and then adyng it to 9 8 5 3 3 1 2. And so commeth 9 8 5 3 3 2 1, whiche roote is 3 1 3 9. From whiche I must abate 3. And then remaineth 3 1 3 6, whiche is the full nomber and Square of the one armie. And hath for his roote 5 6.

For as here is one onely quantitie omitted, so the firste nomber, whiche in other questions of immediate equations, was the verie roote, in these interrupte equations is a rooted nomber, and is here a square nomber: whose roote therfore, I have drawn accordingly. And so triall of this woorkke 5 6, in square maketh 3 1 3 6, from whiche if you abate 4, there will reste 3 1 3 2. Again if you adde 1 0, there will rise 3 1 4 6. And those 2 numbers multiplied together, doe make 9 8 5 3 2 7 2, as the question intendeth.

Master. This you see, what use is in these equations, yet are there many other equations, whiche here be not spoken of: but here after you shall haue moare largely declared, if you shewe your self diligence in this parte.

And one question I will propounde, as stye with out woorkke for breuenesse, that you maie see there is moare behinde. There is a nomber whose Square abated by 1 6, and the firste nomber augmented by 8, and then bothe the multiplied together, will byng for the 2 5 6 0.

Scholar. I will proye the woorkke of it. And therefore suppose the firste nomber to be 1 2 3. Then is his square 1 2 3, whiche abated by 1 6, leueth 1 2 4 9. and the nomber augmented by 8, yeldeth 1 2 4 9. These 2 numbers multiplied together, will make 1 2 4 9, 1 2 8 9, byng euall to 2 5 6 0.
The Arte

1. $x = 16.9$
1. $z = 8.9$

\[ 1.\,\text{c} \quad 16.\,\text{z} \]
8. $z = 128.9$

\[ 1.\,\text{c} \quad 8.\,\text{z} \quad 16.\,\text{z} \quad 128.9 \]

And addyng 128.9 on bothe sides of the equation, it will be $1\,\text{c} + 8\,\text{z} = 16\,\text{z} = 2688.9$

Againe addyngc. 16. $\,\text{z} \quad 16\,\text{z} + 2688.9$

Master. Where at this you now?

Scholar. I see no shifte, but other to leuake it, as it is, 2. numbers equalle to 2. other els to make 1. number equaale to 3. And all that is aboue my cunninge. For hetherto I have learned noe rule for any of them bothe. So that I can not geffe, what the firste number might bee.

Master. The number is 12. And his Square is 144. from whiche if you abate. 16. it will bee 128. And if you add. 8. to 12. it will yeld. 20. Then multipliynng. 128. by. 20. the commye will be 2560. as the question declared.

But to put you out of doubte, this equation is but a trifle, to other that bee untouche. And yet I will tourne this equation a little, to giue you some light in it, and other soche. As here.

\[ 1.\,\text{c} \quad 16.\,\text{z} \quad 2688.9 \quad 8.\,\text{z} \]

where you see. 1. $\,\text{c} \quad$ equaale to. 3. other numbers. And is it not certaine to you, that this equation is true?

Scholar. Yes. I am assured thereof.

Master. And yet to avoide doubtfulness the more trye it by resolution, accoumpynge. 12. of 1. $\,\text{z} \quad$

Scholar. Where. 12. is. 1. $\,\text{z} \quad$ there. 1. $\,\text{z} \quad$ is. 144. and. 1. $\,\text{c} \quad$ is. 1728. Whiche. 1728. must bee equaale to 16. $\,\text{z} \quad$
of Cozike nombers.

16. $2^3$ (that is, 192) and to 2688. saue that you must abate $8^3$, that is 1152. Now if I adde 192 to 2688 it will make 2880, out of which abatynge 1152, there will remaine 1728, wherby I see the equation is inste.

Master. Then you see that the equation is true. And can you doubt, that any number, which is equalle to a Cubike nomber, hath in it a Cubike roote?

Scholar. It must needs bee a Cubike nomber, that is equalle to a Cubike nomber: and therefore must needs have a Cubike roote: although I knowe not how to extrate that roote.

Master. Likewise, when I saie:

$8 \cdot X \cdot X \cdot X = 12 \cdot X \cdot X \cdot X - 128 \cdot X$. It is certaine, not onely that, $12 \cdot X \cdot X \cdot X - 128 \cdot X$, containeth in it as moche as $8 \cdot X \cdot X \cdot X$, but that the $8$ parte of it is a $X \cdot X \cdot X$ nomber, and hath a $X \cdot X \cdot X$ cubike roote.

And farther it is manefeste, that as every $X \cdot X \cdot X$ nomber, dooth containe in it certaine $X \cdot X$ nombers exactly, so if any nomber be anned with those Surfolides (as here in this example are set 128) it is of necessitie, that that 128, must containe in it certaine Surfolides exactly.

So if $8 \cdot X \cdot X = X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X$, then it must needs be that the $8$ parte of this compounde nomber shall be a $X \cdot X \cdot X$ nomber. And also that the $X \cdot X \cdot X$, with the other numbers following doeth containe a certain number of $X \cdot X$ nombers. And the $X \cdot X \cdot X$, in like sorte includeth a number of $X \cdot X \cdot X$ nombers.

And laste of all: $3125 \cdot X$ doeth comprehend certain Cubike nombers exactly.

In like sorte, when we saie, that $1 \cdot X \cdot X \cdot X = X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X$, it is cleare that $1 \cdot X \cdot X \cdot X$, is equalle to $X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X$. All this compounde number is a Surfolide, and hath a $X \cdot X \cdot X$ roote. And $X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X \cdot X$ includeth certaine Cubes. And so
The Arte

doeth. 9.9. containe exactly. 1.2. 0.2moare.

But of these and many other berie excellent and
wonderfullle woorkes of equation, at an other tyne I
will instructe you farther, if I see your diligence ap-
plied well in this, that I haue taughte you.
And therefore here will I make an
Exande of Cobiike numbers,
for this tyne.
Of Surde numbers, in diverse sortes
And firste of Surde numbers
uncompounde.

Now that you have somewhat learned the art of Coskie numbers, with the rule of
equation, it seemeth good time
and apte place, to teache you
the art of Surde numbers, whiche
are diverse in name, acording as there are diverse
natures of roote, which may
give them name.

For generally, a Surde number is nothing els, but
soche a number set fo a roote, as can not be exprested
by any other number absolute.

As the Square roote of $10,02$ of $8$, of any number,
that is not square. Likewise, the Cubike roote of $4,02$
of $5,02$ of any number that is not Cubike. So the Zen
Zizenzieke roote of $8$, $12$, $02$, $02$ of any number that
is no Zenizenzieke, is called a Surde number. And in
like maner, any other roote of any number, that hath
noe soche roote, dooth cause that number to be a Surde
number.

For if you see those signes annered with numbers,
that hath soche roots, those numbers are not Surde
numbers properly, but sette like Surdes. As the Square
roote of $4$, $02$ of $9$, $02$, $25$, &c. The Cubike roote of $8$, $27$,$02$, $125$, &c. whiche sometymes is used fo apte wozke,
as you shall see here after.

Of Numbereation.

The numeration of the dooth consiste, in know-
ledge of their figures, whiche partly be declared
before. But their common and peculliate signes
are these, $\sqrt{\cdot}$, $\sqrt[\cdot]{\cdot}$, $\sqrt[\cdot]{\cdot}$. Although there maie be moare
$119$. varieties
varieties: Yet these for this tyme maie suffice.

The firste, that is, √. is customeably set, to signifie a Square roote. As this, √ 5. betokeneth the Square roote of 5. And √ 12. is the Square roote of 12. Howbeit many tymes it hath with it, for the moare certaine the Cosike signe, ×. And is written thus, √ × 20. the Square roote of 20. And √ × 56. the Square roote of 56.

The seconde signe is anwered with Surde Cubes, to expresse their rootes. As this, × × 16. Whiche signifieth the Cubike roote of 16. And × × 20. betokeneth the Cubike roote of 20. And so for the. But many tymes it hath the Cosike signe with it also: as × × 25. the Cubike roote of 25. And × × 32. the Cubike roote of 32.

The thirde figure doeth repesente a zenzizenzike roote. As × 12. is the Zenzizenzike roote of 12. And × 35. is the Zenzizenzike roote of 35. And like wares if it haue with it the Cosike signe, × × 24. As × × × 24. the Zenzizenzike roote of 24. And so of other.

Scholar. It were against the reason, to seke reason for those signes, whiche be set voluntarily to signifie any thyng; although some tymes there bee a certaine apte conformitie in soche thynges. And in these figures, the number of their minomes, seemeth disagreeable to their order.

Master. In that there is some reason to bee heeded: for as √. declareth the multiplicatie of a number, ones by it selfe: So ×. representeth that multiplication Cubike, in whiche the roote is repesented thysse. And ×. standeth for × ×. that is 2. figures of Square multiplication: and is not expressed with 4. minomes. For so should it seene to expresse moare then 2. Square multiplications. But of voluntarie signes, it is inough to knowe that this thei doe signifie. And if any manne can diuise other, moare cause of ater in se, thei maie well be receiued.

But
of Surde nombers.

But concerning the numeration of Surde nombers this shall you marke: that when any compounde signe is putte before a number, whiche hath any roote, that roote be expressed by parte of that signe, that number is not absolutely so to bee expressed, onlesse it bee for ease or aptnesse in worke. As. √3, 3, 36, whiche betokeneth the senzienzike roote of 36. Seyng it is well known, that 36 hath 6, for 6 is Square roote, it were more apte expressyng that number thus. √3, 6, that is the square roote of 6.

Otherwaies, if the number that followeth the signe, have a roote agreeable to that signe: it is noe Surde number. As. × 5, 16, 15, 4, and is noe Surde number. So. 16, 5, 5, and neadeth not to bee written in Surde forume, excepte it bee for aptnesse of worke. And by this male you judge of all other, as they come in use.

Scholar. If this bee all that is requisite to numeration, I prate you proceed to addition. For that is next in order.

Master. That is the common order. Nowbereth in vulgar fractions, you remember that multiplication and division, are set before addition and subtraction: because of the easier soures of worke in multiplicacion and division. And so in these Surde nombers, because the workes of multiplication, and of division, be not onely more easie, then the workes of addition, and of substracion, but also be requisite to them, therefore will I begin with them, and so come to the other.

Of Multiplication.

Multiplicacion in Surde nombers uncopounde hath noe difficultie, if they be of one denomination: els must they be reduced to one denomination: and that by multiplicacion, according to their signes.

But where noe reduction needeth, you shall multylique
The Arte

tiplie the numbers together, and lette their common signe before the number, that resultth of that multiplication.

Examples of Square Surdes.

If you will multiplie $\sqrt{3} \cdot 15$ by $\sqrt{3} \cdot 26$, it will make $\sqrt{3} \cdot 390$.
So $\sqrt{3} \cdot 32$ multiplied by $\sqrt{3} \cdot 48$ dooth make $\sqrt{3} \cdot 1536$.
And $\sqrt{3} \cdot 56$ multiplied by $\sqrt{3} \cdot 21$ doth yeede $\sqrt{3} \cdot 1176$.

Howbeit some tymes it happenthe, that the number, which is made by that multiplication, is a number absolute, and not a Surde number.

Examples of soche as make numbers Absolute.

\[
\begin{array}{c}
\sqrt{12} \\
\sqrt{3} \\
\hline
\sqrt{36}. \text{that is } 6.
\end{array}
\quad
\begin{array}{c}
\sqrt{48} \\
\sqrt{3} \\
\hline
\sqrt{144}. \text{that is } 12.
\end{array}
\]

\[
\begin{array}{c}
\sqrt{12.\frac{1}{2}} \\
\sqrt{4.\frac{1}{2}} \\
\hline
\sqrt{56}.\frac{1}{2}. \text{that is } 7\frac{1}{2}.
\end{array}
\quad
\begin{array}{c}
\sqrt{28.\frac{1}{2}} \\
\sqrt{7.\frac{1}{2}} \\
\hline
\sqrt{207}.\frac{1}{2}. \text{that is } 14\frac{1}{2}.
\end{array}
\]

\[
\begin{array}{c}
\sqrt{240} \\
\sqrt{15} \\
\hline
\sqrt{3600}. \text{that is } 60.
\end{array}
\quad
\begin{array}{c}
\sqrt{325} \\
\sqrt{13} \\
\hline
\sqrt{4225}. \text{that is } 65.
\end{array}
\]

And generally when any number is multiplied by an other, if the proportion betwene those 2 numbers bee represented by a Square number, as by $4 \cdot 9 \cdot 16 \cdot 25$, &c. then doe thet make a Square number by their multiplication.

Examples
of Surde numbers.

Examples of Cubike rootes.

\[
\begin{array}{ccc}
\sqrt{2} & 9.1 & \sqrt{7} \frac{1}{2} & \sqrt{256} \\
\sqrt{3} & 1.2 & \sqrt{5} & \sqrt{190} \\
\end{array}
\]

\[\sqrt{1.092} \quad \sqrt{5.5} \quad \sqrt{190} \]

Examples of soche as make Absolute numbers.

\[
\begin{array}{ccc}
\sqrt{5} & 4.3 & \sqrt{686} \\
\sqrt{32} & & \sqrt{4} \\
\end{array}
\]

\[\sqrt{1728} \text{ that is } 12. \quad \sqrt{2744} \text{ that is } 14\]

\[
\begin{array}{c}
\sqrt{486} \\
\sqrt{96} \\
\end{array}
\]

\[\sqrt{46656} \text{ that is } 36\]

Examples of zenzizenzike rootes.

\[
\begin{array}{ccc}
\sqrt{15} & \sqrt{24} & \sqrt{162} \\
\sqrt{7} & \sqrt{26} & \sqrt{32} \\
\end{array}
\]

\[\sqrt{105} \quad \sqrt{5304} \quad \sqrt{5184} \text{ that is } \sqrt{72}\]

\[
\begin{array}{c}
\sqrt{7} \\
\sqrt{4} \\
\end{array}
\]

\[\sqrt{5} \frac{1}{2} \text{ that is } 2 \frac{1}{2} \quad \sqrt{324} \text{ that is } \sqrt{18}\]

Examples of zenzizenzike rootes
that make absolute numbers.

\[
\begin{array}{cc}
\sqrt{32} & \sqrt{128} \\
\sqrt{8} & \sqrt{32} \\
\end{array}
\]

\[\sqrt{256} \text{ that is } 16. \quad \sqrt{4096} \text{ that is } 64. \quad \text{Err.} \quad \sqrt{288}\]
But here is to bee noted, that if you would multiplie any Surde number, 
by an absolute number, or any Surde number of one denomination, 
by a Surde number of another denomination: you must firste reduce that 
Absolute number to the like denomination. And so 
must you reduce the 2. Surde numbers to one denomina-
tion.

And because that this woorke doeth serue often in 
this arte, and that in diversse woorkes, I will set here 
the arte of reduction.

Of reduction in Surdes.

Reduction in Surdes, is the bringynge 
of sundrie denominations unto one. 
Whiche in absolute numbers is this 
dozen. You shall multiply the abso-
lute number, according to the signe 
of the Surde, and then sette before it 
the like signe. So that if you would 
double \( \sqrt{2} \), that is to saie, if you would multiply 
it by 2, you must firste multiply that. 2. squarely, and 
then multiply those numbers together. That is to 
saye, you shall multiply \( \sqrt{2} \cdot 2 \) by \( \sqrt{2} \cdot 4 \), and so is 
it doubled.

Likewise, to triple any Square Surde, is to multi-
plie it by 9. And so to quadruple any square Surde, is to 
multiply it by 16. And so forthe.

But if you double any Cubike number, you shall 
multiply it by 8. that is the Cube of 2. And so if you 
would triple a Cubike roote, you muste multiply it by 
27. And if you would quadruple it, you shall multiply
Of Surde numbers.

It by. 6 4. And so of other like woorkes.
Again, if you will double any zenjizenrike roote, you must multiply it by 16. And if you will triple it, you shall multiply it by 81. And so if you will quadrisple it, you must multiply it by 256. And in like manner evermore, for the number absolute, you shall set his zenjizenrike number. Like as in Squares, for any number absolute, you shall set his square. And in Cubes you shall take his Cube.

Scholar. This is plaine enoughe: yet I praise you put an example or two, of eche kinde.

M przez. Take these examples for square rootes.

\[
\begin{array}{ccc}
\sqrt{38} & \sqrt{5} & 128 \\
2 & 6 & 12 \\
\sqrt{152} & \sqrt{8} & 4608 \\
& 469976 \\
\end{array}
\]

Examples in Cubike rootes.

\[
\begin{array}{ccc}
\sqrt[3]{52} & \sqrt[3]{163} & \sqrt[3]{4806} \\
2 & 5 & 8 \\
\sqrt[3]{416} & \sqrt[3]{20375} & \sqrt[3]{2460672} \\
\end{array}
\]

Examples in zenjizenrike nombers.

\[
\begin{array}{ccc}
\sqrt{69} & \sqrt{251} & \sqrt{1385} \\
2 & 4 & 5 \\
\sqrt{1104} & \sqrt{64256} & \sqrt{2250625} \\
\end{array}
\]

Scholar. This I perceue well. But now in Surde numbers of diverse denominations, what the order of reduction is, I praise you to set forth with some examples.

M przez. These examples with their declaration, make sufficiently serve for the use, if I would multiply. \(\sqrt[3]{12}\) by \(\sqrt[3]{5}\). I must firste multiply the number of one signe, accordingly to the signe of the other.
The Arte

number, and so alter them both. Whiche woorke is like the reduction of fractions, to one common deno-
nomination. As here I wulle multiplie. 5. Cubikely, and 12. must be multiplieid squarely, and then shal I addo
bothe signes in one, for their common signe. So shal I have for them the \( \sqrt[3]{5 \cdot 2} \) roote of 144, to be multi-
plieid by the zenzice bike roote of 125. And so wil there come of \( \sqrt[3]{5 \cdot 2} \) 144.
that multiplication, the zenzicke \( \sqrt[3]{5 \cdot 2} \) 125.
ike roote of 18000. As here by \( \sqrt[3]{5 \cdot 2} \) 18000.
example doeth appeare.

Likewise if I would multiplie, \( \sqrt[3]{5 \cdot 2} \), by
\( \sqrt[3]{5 \cdot 2} \). I shal firste multiplie, 250, Cubikely, and it
will bee 17625000. And 34. must I multiplie zenzicke,
ike, and it will yeilde, 1336336. Wherefore
multiplieing them together, and addyng thereto the
common denomination, it will bee the \( \sqrt[3]{5 \cdot 2} \) rootes
of 2088025000000.

This woorke is aptly representeid in signe, after
this sosse. And then shal you multiplie eroke wares
the number of the one, by the signe of the other. And
so make you dooe in all other like numbers, of diverse
denominations.

This reduction doeth serve for any other woorke,
as well as for multiplication.

Of Division.

Division is as easie as multiplication. For
in it there is no regard had to the signes.
But the one number divided by the other
as if they were numbers absolute. And then
the firste signe added to the quotiente. For
the more lighte and certaine, I have set here, exam-
pies of eche sosse.

And
of Surde nombers.
And first examples of square rootes.
\[ \sqrt{72} : (\sqrt{9}, \text{that is}, 3) \quad \sqrt{128} : (\sqrt{32}) \]
\[ \sqrt{8} : \quad \sqrt{4} : (\sqrt{2}) \]
\[ \sqrt{457} : (\sqrt{21}) \]

Examples of Cubike rootes.
\[ \sqrt{96} : (\sqrt{24}) \quad \sqrt{1664} : (\sqrt{32}) \]
\[ \sqrt{4} : \quad \sqrt{32} \]
\[ \sqrt{5624} : (\sqrt{74}) \quad \sqrt{76} \]

Examples of zenizenzike rootes.
\[ \sqrt{54} : (\sqrt{9}, \text{that is}, 3) \]
\[ \sqrt{6} \]
\[ \sqrt{286} : (\sqrt{6}) \quad \sqrt{5892} : (\sqrt{109}) \]
\[ \sqrt{42} \]

And this male suffice for Division. The prose of it is by the contrary kinde. For Multiplication proceuth Division: and Division triceth Multiplication.

Scholar. All this is case inough to remember.

Of Addition.

Mster.

Addition is not so case, but hath diverse The fisste varieties of worke, as anon shall appere. forme of Whereof the fisste is as case as can bee. Addition. For it requireth onely the signe of addition. As if I would addde \( \sqrt{12} \) to \( \sqrt{19} \).
The Arte

\[ \sqrt{26} \] thall set it thus. \[ \sqrt{26} \] \[ \sqrt{12} \] And so
\[ \sqrt{20} \] put unto. \[ \sqrt{16} \] maketh. \[ \sqrt{54} \] \[ \sqrt{20} \] This some scruteth chiefly for roots of diverse names. As \[ \sqrt{7} \] \[ \sqrt{5} \] \[ \sqrt{20} \] \[ \sqrt{7} \] \[ \sqrt{30} \] where
\[ \sqrt{30} \] is added to \[ \sqrt{7} \] \[ \sqrt{20} \] And so of all other.

The seconde some is not so easie; and yet many times it is no more certaine. And this is the order of it.

You shall doe one your. 2. numbers, that you would adde together, for seyng that they be of one denomination. Then shall you adde in plaine some, their numbers together, puttynge thereto the signe of the roote. And kepe that as a parte of the addition. Again you shall multiplie the 2. firste numbers togethers. And their totalle you shall multiplie by. 4. And before that shall you sette the signe of the roote. And it shall stande as the seconde parte of that addition. So that those 2. partes, shall be added with the signe ... And then is the woozke canded. Example hereof. I would adde the 2. firste somes, that is, \[ \sqrt{12} \] to. \[ \sqrt{26} \]. Wherfor I
set them thus. And then doe
\[ \sqrt{26} \] \[ \sqrt{12} \]
\[ \sqrt{38} \] \[ \sqrt{12} \]
\[ \sqrt{312} \] \[ \sqrt{4} \]
\[ \sqrt{1248} \]
I adde the bothe plainly togethers, and the make. \[ \sqrt{38} \] which I set by, as one part of the addition. Then doe I
multiplie \[ \sqrt{26} \] by. \[ \sqrt{12} \] and there riseth. \[ \sqrt{312} \] which
I must double, or multiplie \[ \sqrt{38} \] \[ \sqrt{1248} \] by. 2. And therfor seyng the
woozke is in square rootes, I set the square of 2. with the signe of. \[ \sqrt{38} \] \[ \sqrt{2} \] and then multipliyng theim togethers, I have. \[ \sqrt{1248} \] which is the seconde parte of the roote. Wherfor seythe adde yng those 2. partes togethers, with the signe ... there commeth. \[ \sqrt{38} \] ... \[ \sqrt{1248} \] as the totalle of that addition.

Scholar. As me thinketh, the firste some of addition,
of Surde nombers.

It serveth better for these nombers, then this se-
conds forme. For it is moare easie to use, in any kind
of woork, and moare speedily done: and it serveth that
this laste nomber, is moare obscure then the firste.

Walter. Yet is this woork good, and very neces-
sarie. For in these nombers, and sothe other like, it
serveth onely (as appereth) to alter the state of the no-
bbers, whereby the maie bee commensurable, with o-
ther, then they were before that alteration. But in
some nombers, and that very many, it reduceth them
to one simple soure of roote. As by the examples fol-
lowyn you shall perceiue.

An exaple.

<table>
<thead>
<tr>
<th>( \sqrt{28} )</th>
<th>( + ) ( \sqrt{7} )</th>
<th>( \sqrt{28} )</th>
<th>( + ) ( \sqrt{7} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{28} )</td>
<td>( \sqrt{7} )</td>
<td>( 28 )</td>
<td>( 7 )</td>
</tr>
<tr>
<td>( \sqrt{196} )</td>
<td>( \sqrt{4} )</td>
<td>( 196 )</td>
<td>( 14 )</td>
</tr>
<tr>
<td>( \sqrt{784} )</td>
<td>( \sqrt{63} )</td>
<td>( 784 )</td>
<td>( 14 )</td>
</tr>
</tbody>
</table>

The same example other
wases woughte.

\( \sqrt{35} \) \( + \) \( \sqrt{784} \)
\( \sqrt{35} \) \( + \) \( 28 \)
\( \sqrt{35} \) \( + \) \( 784 \)

Where firste I have set fothe, 2. examples of one
addition, that you may see the agreenceme of the both
And firste I would adde, \( \sqrt{28} \) with \( \sqrt{7} \). where-
foxe I dooe lync. 28. and 7. in one fomme, which I
set a parte, as the froste portion of the addition. Then
I doe multiply. 28 by 7. And thereof commeth. 196.
which is a square nober, and hath. 14. for his roote.
So that I maie use now. 2. workes. For other I maie
continue my woorkke, as I have done (agreable to the
firste exaple) in multiplying that, \( \sqrt{196} \) by \( \sqrt{4} \).
(whiche is but doubling) and so there cometh, \( \sqrt{784} \)
whiche
The Arte

which is a number absolute: because it hath a root, according to his signe, which the roote is 28. and must be set for. √ 784.

Now in the second woorke, because the first multiplication of 28 by 7 doeth make a square number, I doe take the root of that number for it: being it is all one thing to see. √ 196, and. 14. soz. 14. is the root of 196. And then having the root, I must double it, according to the rule, or multiply it by 2, and there of commeth. 28. Whiche I shall adde with 35. And so haue 1.63. whose roote containeth the addition of √ 28. and √ 7.

Scholar. This woorke semeth strange: and farthest from common reason, of all other woorkes in this arte.

Master. I mighte easily by demonstration make you to perceive as moche reason in this woorke, as can be in any: for it dependeth of the. 38. Theorem of the pythagorean. But haste of other businesse, maketh me to omit the demonstration at this tyme, which shortly you shall haue, for all the equations, and other woorkes likewaies.

But for this presente tyme, it shall be sufficiente to woorke an example in rationall numbers, as if theier wer surde numbers: that therby you maie perceive the order, and the truth of the woorke.

Wherefore I take these twoo numbers, √ 36. and √ 49. to bee added together. Where I doe firste adde the twoo numbers plainely together: And therwith make 85. soz. the firste parte of the addition. Then doe I multiplye. 49 by. 36. and there riseth. 1764. Whiche is a square number. And therefore maie I bee. 2. woorkes, as you see. In the firste I multiplye that Square number by. 2.02 by √ 4. which is all one: and there doeth amounte. 7056. a Square number also, whose roote is 84.

The
### Of Surde numbers.

#### The firste forme.

<table>
<thead>
<tr>
<th>$\sqrt{36}$</th>
<th>$\sqrt{49}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{49}$</td>
<td>$\sqrt{36}$</td>
</tr>
<tr>
<td>$49$</td>
<td>$36$</td>
</tr>
<tr>
<td>$294$</td>
<td>$85$</td>
</tr>
<tr>
<td>$117$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{1764}$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{4}$</td>
<td></td>
</tr>
<tr>
<td>$7056$</td>
<td></td>
</tr>
</tbody>
</table>

#### The seconde forme.

<table>
<thead>
<tr>
<th>$\sqrt{36}$</th>
<th>$\sqrt{49}$</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>$294$</td>
<td>$147$</td>
</tr>
<tr>
<td>$85$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{1764}$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td></td>
</tr>
<tr>
<td>$84$</td>
<td></td>
</tr>
</tbody>
</table>

That is, $\sqrt{169}$.

**Dr. 13.**

In the seconde woozke I take the roote of $1764$, which is 42 and doublyng it, I haue 84, agreable to the other woozke. Then doe I sette those 2 numbers downe with $\sqrt{\text{sign}}$, and putte to them the signe $\sqrt{\text{sign}}$, in token that I muste take the roote of that compounde number: and not of any one parte of it.

Scholar. That haue I marked well: Fo $85$, hath no roote, nother $84$, hath any roote. But $85 + 84$ that is, $169$, hath $13$, for his roote.

And so I see, that the roote of $36$, which is 6. And the roote of $47$, that is, 7, beying bothe added togethe will make $13$, that is the roote of $169$.

Master. Yet one other forme of easie woozke, I will shewe you, which is bothe pleasaunte and pro: stable: But is not generalle, for it serveth onely fo: numbers commensurable, I meane suche numbers, as by forms, one common diu: so, made bee brought into Square numbers. With whiche numbers, you shall woozke thus.

**Dr. 7.**  Firste
The Arte

Firste divide theim by the common diuisor: and set for them their rootes. Then add those 2 rootes together, and multiply it squarely. And that square being multiplied by the common diuisor, will brynge for the Square of both the rootes. As here foloweth in example.

Where I would adde $\sqrt{384}$ unto $\sqrt{150}$ which numbers I doe examin, til I maie finde their commo, and leathe diuisor, whiche here is 6. Then diuiding them by that 6, I have for 384 a square number 64. And for 150 I have an oother square, that is 25. Of whiche bothe squares I set downe the rootes: and the common diuisor also. Then doe I adde bothe rootes together, and thereof commeth 13, whose Square is 169, that I doe multyplic by 6, whiche is the commo diuisor, and it will bee 1014, whose roote doeth contain bothe the rootes before named. As you shall see it proed anon by Subtraction.

Scholar. In the meanes season I consider, that one of these forms, maie confirme the other. And therefor is if I woozke this laste example, by one of the other forms, and finde thesame totall, it muske needes be that the woozke is good. Whiche I prooe thus.

Firste settynge downe the numbers, in form of the easieste Addition. And then addynge them together, I finde 534, whiche I sette a side, as one parte of the number, that I doe seke for.

Then doe I multiply the 2 numbers together, and thei make 57600, whiche I doe multiply again by 4. And there riseth 230400, being a square number, and hath 480 for his roote. Wherefore I let
of Surde numbers.

\[
\begin{array}{c|c}
\sqrt{384} & \sqrt{150} \\
384 & 384 \\
150 & 150 \\
19200 & 534 \\
384 & \\
57600 & 4 \\
230400 & \\
\end{array}
\]

\[
\sqrt{534} + \sqrt{230400}.
\]

\[
\text{Or: } \sqrt{534} + 480.
\]

That is, \( \sqrt{1014} \).

1014, agreeably to the other woorkke. Wherefor I judge them bothe to be good.

Master. You might have wroght this woorkke otherwise, because the firste number, that is the square number, was the multiplication of two numbers.

Scholar. Then I perceive, I might have taken the root of it, which is 240, and doublyng it, I should have 480. As I had in the other woorkke. And so all doe agree in one.

But my chief doubt is, how to knowe those numbers that bee commensurable: For if I shall stande long in searching for that, I might sooner woorkke the other soume of woorkke, then to make that trialle of commensurablenesse.

Master. The easieste waie is, to divide the greatter number, by the lesser, as if thei were bothe nombres absolute: the quotiente will declare their Squares commensurable.

As if you doubt the, whether 384 and 150 bee commensurable, divide 384 by 150, and the quotiente will be \( \frac{4}{3} \), that is \( 1 \frac{1}{3} \). Then divide whiche of the 2. firste numbers you like, by his like number in the quotiente: And the common diuido2 will amounte. So if you di-
The Arte

Nide the greater number, 384, by the greater number in the quotient, which is 64, you shall finde the new quotient 6. Whiche 6. is the common number. If you divide 150 by 25, the common number 6. will be the quotient.

But if the quotient be a whole number, and no fraction, and be a square number, then is it the lesser square. Wherefore if you divide the lesser number of the 2. by the quotient, the common number will appeare in the seconde quotient. And then if you divide the greater of the 2. numbers, by that common number, his quotiente will the we you the other square.

And if it so happen, that the quotient of the first division be not a square number, then are those numbers incommensurable.

So \( \sqrt{32} \) and \( \sqrt{128} \) bee commensurable: and the quotiente of their division is 4. Whiche is the lesser square. And 8. appeareth to be the common number. And the greater square is 16.

Howbeit by this number it maye easily bee espied, that some numbers maye be resolved, into more squares then one. As these 2. numbers, being divided by 2 dooe give 16 and 64. And being divided by 8, the bying for the 4. and 16.

But for their addition, what Squares so ever you take, that redounds by one common diuisio, the triall will be like, and the roote one.

Scholar. If ye let me prowe that varietie.

Master. Then prowe it in lothe numbers, where you may finde moare varietie. As these bee \( \sqrt{288} \) and \( \sqrt{1152} \).

Scholar. If I divide 1152, by 288, the quotiente will bee 4. Whiche I must take for the leaste square. Then by it I divide 288, and the quotiente will be 72. as the common diuisio. By whiche if I divide 1152, there will life 16. as the seconde square. Then let the
of Surde nambers.

the nbors in oeder thus. \[ \sqrt{1152} - \frac{1}{1} - \sqrt{288}. \]

And under 1152 I set the one Square. 16. And under 288 I putte the other square. 4. And under eche of them his roote. Then adde I the Rotes together, whiche maketh 6. whose square is 36. And that byeng multiplied by 72, the common number, doeth yeld 2592. whose roote doeth containe bothe the other. 2. rootes by addition.

But now how shall finde any other squares in those nbors, to make any farther trial, I knowe not.

Halter. Divide alwaies one of the numbers, by some square nbner, that will parte it exactely, without any remainer. Andmarke the quotiente. For by it shall you divide the other nbner, and if the quotiente in that last division, be a square number, then have you your purpose. Else make you prowe with an other Square number.

Scholar. I understande you. And therefore in these numbers, I will make triale with 9. by whiche I divide 288. And finde the quotiente, 32. Then by the same 32, I divide 1152, and the quotiente is 36. So haue I 9 and 36, for the 2 squares, and 32, for the common divisor. Wherfore I set the nbors in order as they ought. And under them I place the 2. square numbers with their rootes. Then addeinge the rootes together, I finde 9, whiche I multiplye square, and it yeldeth 81, that 81. I doe multiplye by the common number, 32, and there amounteth 2592. As it did before in the other worke. Wherby I perceiue that these workes doe confirm one an other.
And therefore I will prove, how many varieties of this woork, I may finde in these nombers.

And for my purpose, I will divide the letter of the 2. nombers, by as many Squares as I can, so the leaseth to be the readiest waie. And firste I prove with 16. And so the quotient is 18. by whiche 18. I divide 1152. and the quotiente is 64. which is a square nomber. So that I haue that variete more.

Then again I prove with 25. But I see, that will not frame. Wherefore I assaye with 36. And finde the quotiente 8. by whiche I divide the greater square, and the quotiente is 144. a square number also. And therefore I note that for an other variete.

Thirdly, I prove with 49. but that will not agree. Then attempete I with 64. And that serveth as euil. Perete that I assaie 81. 100. and 121. but none of them will divide 288. Wherefore I passe unto 144. which is twice contained in 288. by that 2. I divide 1152. and finde the quotiente 576. which is a Square number also. And so haue I 3. other varieties beside the 2. former woorkes: which 3. varieties, for my re-membraunce I set done, thus.
of Surde nombers.

\[
\begin{array}{c|c}
\sqrt{1152} & \sqrt{288} \\
\hline
64 & 16 \\
18 & 8 \\
12 & 4 \\
\hline
144 & 36 \\
8 & 12 \\
18 & 6 \\
\hline
324 & 8 \\
\hline
1152 & 144 \\
\hline
\sqrt{2592} & \\
\end{array}
\]

Master. Then for to gratifie you, I will sette done 2, other number with 6 varieties. Which maie seeme to suffice for this worke, without more examples. And because you know the order to trie the I will sette them done without any explication, other declaration. As here you see.

\[
\begin{array}{c|c}
\sqrt{28800} & \sqrt{7200} \\
\hline
14400 & 3600 \\
2 & 120 \\
180 & 60 \\
\hline
3600 & 900 \\
3 & 60 \\
90 & 30 \\
\hline
32400 & 8100 \\
2 & \\
\hline
\sqrt{64800} & \sqrt{64800} \\
\hline
\sqrt{28800} & \\
\end{array}
\]
\[ \begin{array}{c|c|c|c}
\sqrt{28800} & \sqrt{7200} & \sqrt{28800} \\
1600 & 400 & 900 \\
18) & 40 & 32) \\
& 20 & 30 \\
60 & 45 & \\
60 & & 45 \\
\hline
3600 & & 2025 \\
18 & & 32 \\
\hline
28800 & & 4050 \\
36 & & 6075 \\
\hline
\sqrt{64800} & & \sqrt{64800}
\end{array} \]

Scholar. This variect is pleasaunte.

Master. I will satisfy your delite better at more lesseare. But yet one thyng moare will I saie, before we can se this sorte of Addition: that if you would adde any roote to it self. As. \( \sqrt{6} \) to \( \sqrt{6} \). \( \sqrt{2} \) to \( \sqrt{10} \). \( \sqrt{10} \). \( \sqrt{0} \), ye shall only quadriple the number: and so have you done.

Scholar. I see good reason in that: For addition of any number to it self, is but doubling that number or multiplication by 2. And that must be done by that quadruplication, as you taught before.

Addition of cubike rootes

Master. Now will I set fo the some examples of addition in Cubike rootes. For the worke is like unto this laste some in Square rootes, saue that the multiplications,
Of Surde nombers.

Triplications, which were Square in that woork, must be Cubike in this woork. And that onely in nombers commensurable. For nombers incommensurable bee added with the signe. ---. without noare woork.

I call soche Cubike rootes commensurable, which be Cubike roote being diuided by any common nomber, will make Custes commensurable numbers in their quotiente. As. \( \sqrt[3]{24} \) and \( \sqrt[3]{81} \) sururable.

Which diuided by 3. doe make 8. and 27. bothe being Cubike numbers. So. \( \sqrt[3]{320} \) and \( \sqrt[3]{135} \) bothe being diuided by 5. doe make 27. and 64. bothe Cubike numbers. Likewises. \( \sqrt[3]{2744} \) and \( \sqrt[3]{1000} \) be commensurable, because thei make 343. and 125. bothe (Sus) Cubike numbers: If thei be diuided by 8.

Scho. I praze you make your examples with these.

Mister. There nedeth noe woordes in this woork it is so like the Addition of square rootes. And therefore marke these examples well.

\[
\begin{array}{c|c}
\sqrt[3]{81} & \sqrt[3]{24} \\
27 & 8 \\
3 & 3 & 2 & 5 & 7 \\
5 & & 125 & & 7 \\
\hline
\sqrt[3]{375} & \sqrt[3]{1715} \\
3 & 5 \\
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt[3]{2744} & \sqrt[3]{1000} \\
343 & 125. \\
8 & 12 & 12 & 1728 & 8 \\
\hline
\sqrt[3]{13824} \\
\end{array}
\]

Do. Scholar.
The Arte

Scholar. Here is noe diversitie, from the forwarde worke, but in setting the Cubike roote, for the square roote. And in multiplying the addition of the 2. rootes Cubike ly.

Mater. That is all. And therefore will I stande noe longer aboute it: But will proseede to another forme of addition, whiche serueth also for Cubike rootes commensurable. The rule is this. Set downe the Cubike rootes, with their common diuisor, and the Cubes that rise therby, and their rootes also. All this you did in this former worke. But now pecullarly in this rule, you shall set downe 3. other numbers orderly, under those 3. former numbers. The firste is the square of that laste Cubike roote: the seconde is the triple of that square: and the thirde is a number resulting of the multiplication of that triple by the other roote.

Then take the 4 extreme numbers, that is those 2 laste numbers, and the 2 Cubes, and adde them together into one somme. And that somme being multiplied by the common diuisor, will make a Cubike number, whose Cubike roote shall containe bothe the firste rootes, whiche you intended to adde. Now marke these examples: and cofere them well with the wor- des of the rule.

\[
\begin{array}{c|c}
\sqrt{384} & \sqrt{48} \\
64 & 8 \\
4 & 2 \\
16 & 4 \\
48 & 12 \\
\hline
48 & 96 \\
\hline
\sqrt{1296} \\
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt{15972} & \sqrt{2592} \\
1331 & 216 \\
121 & 36 \\
363 & 108 \\
1188 & 2178 \\
\hline
4913 & 4913 \\
\hline
9826 & 9826 \\
4913 & 4913 \\
\hline
58956 & 52488 \\
\end{array}
\]
of Surde numbers.

\[
\begin{array}{c|c}
5832 & 2744 \\
9 & 1 \\
324 & 196 \\
972 & 588 \\
10584 & 13608 \\
32768 & 9 \\
\hline
\text{mm.} & 294912
\end{array}
\]

Scholar. In these examples I see, the woordes of your rule observed. For under each Surde Cubike roote, there is set a true Cubike number, which is founde by the common diuision: then followeth the roote of that true Cube: and beside it standeth the common diuision. Then in the fouerthe roome is the Square of the true Cubike roote. And under it his number tripled (as 48 under 16, and 12 under 4) whiche triple being multiplied by the roote of the other side, dooth make the loweste number in that rowe. So 48 multiplied by 2, maketh 96, which is set under the roote. 2 and 12 multiplied by 4, yeldeth 48, which is placed under that 4.

Then those 4 extreme numbers, 64 and 48, 8. 96 these make by addition 216, whiche somme is multiplied by 6, that is the common diuision, and so riseth 1296. whose Cubike roote comprizeth bothe the firste rootes.

Master. The like maie you judge of the other 2 examples. But because you maie understande the certaintie of this woode the better, I have here sette fothe 2 examples of true Cubike rootes, soymed like Surde numbers.

\[\text{Do.}\] \[w'.4096\]
The Arte

\[
\begin{array}{cccc}
\text{w'.} & 4096. & - & \text{w'.} 1728. \\
512. & - & 216. \\
8) & 64. & 6. \\
& 192. & 36. \\
& 864. & 108. \\
\hline
& 2744 & 1152. \\
\text{w'.} & 21952 & \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{w'.} & 19683. & - & \text{w'.} 3375. \\
729 & - & 125 \\
27) & 9 & 5 \\
& 81 & 25 \\
& 243 & 75 \\
\hline
& 675 & 1215 \\
\end{array}
\]

Scholar. I perceiue by cramination of woork in my Tables here, that 4096 is a Cubike number, and hath 16 for his roote. So 1728 is a Cubike number also, his roote is 12, those bothe rootes added together doe make 28. And that 28. is the Cubike roote to 21952. as the firste example would.

And for the seconde example, I see likewaies that 19683. hath 27. for his Cubike roote. And 3375. hath 15. for his roote. And the bothe make. 41, which is the Cubike roote to 74088. according to the woork of the seconde example.

Addition of Master. Seeing you are conveniently instructed, in these numbers, wee will goe in hande with Zenzie like rootes, and their addition whereby is no difference of woork, but onely for the multiplicatio, which must be agreeable to the nature of the numbers, ZENZEZENZEZENZELY. And the reduction by the common division.
of Surde numbers.

For in like form, into zenzizenzike numbers, whè the firste numbers bee commensurable. But if thebe incommensurable, then must the addition be wroughte by the signe—, without any other businesse.

Examples of zenzizenzikes being commensurable.

\[
\begin{array}{c|c|c}
\sqrt{648} & 81 & 256 \\
8 & 3 & 4 \\
8 & 10 & 10 \\
4096 & 8 & 10000 \\
32768 & & 50000
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt{38416} & \sqrt{65536} \\
2401 & 4096 \\
16) & 7 \\
15 & 15 \\
50625 & 16 \\
303750 & 50625 \\
810000 &
\end{array}
\]

In the firste and second examples the numbers are Surdes, but in the thirde example thei are rationall numbers, framed like unto Surdes to the intente that you mighte the better perceiue the forme of the worke. For \(38416\) is a zenzizenzike number, and hath 14. for his rote. \(65536\), is a zenzizenzike number, and hath 16. for his rote. And these 2. rootes do make 30. which is the zenzizenzike rote unto 810000. And therefore make it see truely sayer, that \(\sqrt{810000}\) doth containe the twoo firste rotes.

Scholar. I prate you procede to Subtraction. For all this I doe well perceiue.
The Arte
Of Subtraction.

Master.

Subtraction doeth differ from addition, in little more than the signe ——. Which signe serveth generally, for all numbers incommensurable. And considering there is little difficulty in Subtraction: If you remember well the arte of Addition, I will lightly passe it over in the same examples, that I have wrought in Addition, because it may bee a proofe of that woork: and that woork also a confirmation of this.

Only this shall you obserue in this rule peculiarly: that as in the seconde sozme of Addition, you must addde the rootes together, before you multipliche them. So here you shall Subtracate the letter root, from the greater, before you doe multipliche them.

Example of Subtraction, with ——.
√.12. abated out of √.26. maketh √.26 —— √.12.,
and so of other.

Examples of the seconde forme of Subtraction

<table>
<thead>
<tr>
<th>√.63. —— √.28.</th>
<th>The seconde sozme of that woork.</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>63</td>
</tr>
<tr>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>504</td>
<td>63</td>
</tr>
<tr>
<td>126</td>
<td>28</td>
</tr>
<tr>
<td>91</td>
<td>1764</td>
</tr>
<tr>
<td>√.1764</td>
<td></td>
</tr>
<tr>
<td>√.4</td>
<td></td>
</tr>
<tr>
<td>√.7056</td>
<td></td>
</tr>
</tbody>
</table>

That is, √.7.
√.169
\[ \sqrt{169} = \sqrt{36} \]

\[ \begin{array}{cc}
169 & 36 \\
1014 & 205 \\
6084 & \\
24336 & \\
\end{array} \]

\[ \sqrt{205} = \sqrt{156} \]

That is, \[ \sqrt{49} \].

Scholar. I see in all these examples, you take the same numbers, that you had before in Addition. And first he set the total, out of which you abate one of the numbers that before were added, and the remainder bringeth for thee the other. For in the first of these examples, \[ \sqrt{28} \] is abated out of \[ \sqrt{63} \], and there remaineth \[ \sqrt{91} \]. That is \[ \sqrt{7} \]. For \[ 84 \] taken out of \[ 91 \] leaveth \[ 7 \]. And in the second example, \[ \sqrt{39} \] abated out of \[ \sqrt{169} \], doeth leave remaining \[ \sqrt{49} \].

Walter. The third form of Subtraction, is like the third form of Addition, sauc that we set — — — —. And here we must abate the lesser root from the greater (as I said) before we do multiply that number by itself. As by this example, you may perceive where I do Subtracte \[ \sqrt{105} \] out of \[ \sqrt{1014} \], and the remainder is \[ \sqrt{384} \]. Now marke the woorkke

\[ \sqrt{1014} = \sqrt{105} \]

\[ \begin{array}{cc}
169 & 25 \\
6) & 13 \\
8 & 5 \\
8 & 64 \\
6 & \\
\end{array} \]

\[ \sqrt{384} \]

Here you see all things agree, with the form of Addition, sauc — — — —. For — — — and when I begin to gather the number, that standeth in the middle, which I multiply by itself, and I do not make that number,
The Arte

number, by adding both the rootes together: For so 13, and 5, would make 18, but I abate 5 out of 13, and so there doth remain 8, with which I proceed as I did in Addition. And then commeth so the the remainder.

\[ \sqrt{384} \]

Scholar. I understand it very well. And I praise you that for a proofe, I make bare the other examples of addition. Partly for my exercise, and partly for examination of the former additions, by the contrary kind.

Mater. With good will.

Scholar. Then will I set them, and worke them, as here foloweth.

But firste I will begin with the worke of this last example, after the seconde forme of Subtraction: for a double confirmation of it.

\[
\begin{array}{c|c|c}
\sqrt{1014} & \sqrt{150} \\
1104 & 1014 \\
150 & 150 \\
50700 & 1164 \\
1014 & \\
\sqrt{152100} & \\
\sqrt{4} & \\
\sqrt{608400} & \\
\sqrt{1164} & \sqrt{608400} \\
O: \sqrt{1164} & 780 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\sqrt{1014} & \sqrt{150} \\
1014 & 1014 \\
150 & 150 \\
50700 & 1164 \\
1014 & \\
\sqrt{152100} & \\
\sqrt{390} & \\
\sqrt{2} & \\
\sqrt{1164} & 780 \\
\end{array}
\]

An other forme of the same worke.

That is \[ \sqrt{384} \]

And now here are the variations of the other examples.

\[ \sqrt{2592} \]
Of Surde numbers.

\[
\begin{array}{cc}
\sqrt{2.592} & \sqrt{2.88} \\
36 & 4. \\
72) & 6 \\
& 36 \\
& 72 \\
= & 4 \\
1152 & 1152
\end{array}
\]

\[
\begin{array}{cc}
\sqrt{2.592} & \sqrt{2.88} \\
144 & 16. \\
18) & 12 \\
& 8 \\
& 64 \\
& 18 \\
& 512 \\
& 64 \\
= & 1152 \\
& 1152
\end{array}
\]

\[
\begin{array}{cc}
\sqrt{2.592} & \sqrt{2.88} \\
1296 & 144. \\
2) & 36 \\
& 24 \\
& 21 \\
& 576 \\
& 2 \\
\sqrt{1152} & \sqrt{288}
\end{array}
\]

\[
\begin{array}{cc}
\sqrt{2.592} & \sqrt{1.152} \\
1296 & 576. \\
2) & 36 \\
& 12 \\
& 12 \\
& 144 \\
& 2 \\
\sqrt{1152} & \sqrt{288}
\end{array}
\]

Other examples varied, for proofe of the like. 6 examples in Addition.

Pp.s. \sqrt{64800}
\[
\begin{array}{c|c}
\sqrt{64800} - \sqrt{7200} & \sqrt{64800} - \sqrt{7200} \\
32400 & 8100 \\
2400 & 600 \\
180 & 90 \\
2 & 30 \\
\hline
120 & 60 \\
120 & 60 \\
\hline
14400 & 3600 \\
2 & 8 \\
\hline
\sqrt{28800} & \sqrt{28800}
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt{64800} - \sqrt{7200} & \sqrt{64800} - \sqrt{7200} \\
3600 & 2025 \\
180 & 45 \\
60 & 15 \\
\hline
40 & 30 \\
40 & 30 \\
\hline
1600 & 900 \\
18 & 32 \\
\hline
12800 & \sqrt{28800} \\
16 & \\
\hline
\sqrt{28800} & \\
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt{64800} - \sqrt{7200} & \sqrt{64800} - \sqrt{7200} \\
1296 & 900 \\
50 & 30 \\
36 & 10 \\
\hline
24 & 20 \\
24 & 20 \\
\hline
576 & 400 \\
50 & 72 \\
\hline
\sqrt{28800} & \sqrt{28800}
\end{array}
\]

Subtraction of Cubike rootes commensurable. And therefore I set the examples onely, without any larger declaration.

\[\text{W. 375.}\]
of Surde numbers.

\[
\begin{array}{c|c|c}
\sqrt{375} & \sqrt{81} & \sqrt{1715} & \sqrt{135} \\
125 & 27 & 343 & 27 \\
3 & 3 & 7 & 3 \\
2 & & 4 & \\
8 & & 64 & \\
3 & & 5 & \\
\hline
24 & & 320 & \\
\end{array}
\]

In the seconde forme of another addition of Surde Cubes, you woorke of remember that you added Subtraction numbers together. But for Surde in Subtraction, you shall adde to eche roote seueralie that, that commeth of his owne multiplication, with the other triple. And then shall you Subtrac te the letter number, out of the greater. And the remainder you shall multipie by the common divisor. And so shall you have the roote that remaineth of the Subtraction. As in example.

\[
\begin{array}{c|c|c}
\sqrt{13824} & \sqrt{1000} & \sqrt{58956} & \sqrt{15972} \\
1728 & 125 & 4913 & 1331 \\
8 & 5 & 12 & 11 \\
7 & 7 & 289 & 121 \\
8 & 8 & 867 & 363 \\
343 & & 6171 & 5537 \\
8 & & 216 & \\
6 & & 12 & \\
\hline
2744 & & 584 & \\
\end{array}
\]

\[\text{Pp.} \sqrt{294912} \]
The Arte

Scholar. In all these examples I use the confirmation of the former addittō. And in these laste woorkes, thus I see peculiate from addittō, that the cube is added with the loweste number in that rowe (as in the first example, 2 1 6. is added with 7 2. and maketh 2 8 8: And, 8. is added with 2 1 6. that peldeth 2 2 4.) And then is the lesser abated from the greater (as 2 2 4. from 2 8 8.) And the remainder (whiche there is 6 4.) set in the middle under bothe the restes of numbers. And then is multiplied by the common number, to make the remainder.

So in the first example, the remainder is \\
\[
\text{w}. \ 384.
\]

W. 4 8. is abated out of w. 1 2 9 6. And in the seconde example where \( \text{w}. \ 1 5 9 7 2. \) is subtracted out of \( \text{w}. \ 5 8 9 5 6. \) the remainder is \( \text{w}. \ 2 5 9 2. \) Like, wales in the thirde example, \( \text{w}. \ 2 4 6 9 6. \) is abated out of \( \text{w}. \ 2 9 4 9 1 2 \) leaveth remaining. \( \text{w}. \ 5 2 4 8 8 \)

Master. But now in addition there foloweth 2 other examples, which by subtraction made bee produced thus: as here you see.

\[
\begin{array}{ccc}
\text{w}. \ 2 1 9 5 2 & \text{w}. \ 4 0 9 6 & \text{w}. \ 7 4 0 8 8 & \text{w}. \ 1 9 6 8 3 \\
2 7 4 4 & 5 1 2 & 2 7 4 4 & 7 2 9 \\
8 & 1 4 & 8 & 1 4 \\
1 9 6 & 6 4 & 1 9 6 & 8 1 \\
5 8 8 & 1 9 2 & 5 8 8 & 2 4 3 \\
2 6 8 8 & 4 7 0 4 & 3 4 0 2 & 5 2 9 2 \\
2 1 6 & 2 1 6 \\
8 & 8 \\
\text{w}. \ 1 7 2 8 & & \text{w}. \ 3 3 7 5 & \\
\end{array}
\]

Scholar.
of Surde numbers.

Scholar. I see, in these examples of Subtraction: that the firste number is the totalle, or laste number in addition. And the seconde number, whiche followoth is the number to be abated; and then laste and loweste of all, is the remainder, whiche was one of the firste sommes in addition.

And though there remaine, other examples of xenz, xizenzike numbers, I see no difficulty in them, but that I can woorkke them: As here I have set the sooth.

\[
\begin{array}{c|c}
\sqrt{32768} & \sqrt{648} \\
4096 & 81 \\
8) & 8 \\
5 & \\
5 & \\
625 & 8 \\
\hline
5000 & \\
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt{5000} & \sqrt{1280} \\
10000 & 256 \\
5) & 10 \\
6 & 4 \\
6 & \\
1296 & 5 \\
\hline
6480 & \\
\end{array}
\]

\[
\begin{array}{c|c}
\sqrt{810000} & \sqrt{65536} \\
50625 & 4096 \\
16) & 15 \\
7 & 8 \\
7 & \\
2401 & 16 \\
\hline
38416 & \\
\end{array}
\]

Mister. Seeing you are experte enough in the soe woorkkes of these Surdes uncounde, I wil teache you the like woorkes in condounde Surdes.

Scholar. Is there the noe reduction, nother extracation of rootes, to bee taughte in these uncomounde Surdes?

Mister. As soe reduction, I have taughte you all reade in multiplication, as moche as is required in these numbers.

And soe extracation of rootes, you maie sone understande, that here can be none. For then were thei not Surde numbers. And therefore I laid vnto you before, Pp.13. that
The Arte

that. √ 3. 100. is not a Surde number, although it be written like a Surde number, because it hath a Square roote, according to his signe; and that is. 10. Like wise. √ 256. is no Surde number: for his Square roote is known to be 16.

Scholar. I might have considered as moche, by the definition of Surde numbers, that their rootes can not be assigned in nombers absolute. And therefor I see that. √ 125. is no Surde number, with his Cubike roote is 5. And. √ 256. is a number rationalle, and no Surde number: for his zenzizenikke roote is 4.

Master. But. √ 64. is a Surde number, and yet hath. 64. a Square roote, and a Cubike roote also, but not a zenzizenikke roote, according to his signe. And therefore ought better to be written thus. √ 8.

Scholar. I praeie you to proceed to Surde numbers compoundde.

Of Surde nombers compoundde.

Master.

Vrde nombers compoundde, are made not onely of. 2. 02. 3. or more Surde nombers tr compoundde, but also of rationalle 02 Abstrakte nombers toged with Surde nombers. As. √ 10 —— √ 12 and 8. —— √ 6. like wis. √ 2 0. —— 3. and. √ 4 0.

Compounde Surdes. 

But here shal you marke, that I call compoundde nombers, not onely soche as haue the signe of. ——, but also soche as haue the signe of —— for although in nature of the number. √ 1 0 —— √ 5. be not compoundde, but abated, yet in name he is compoundde, and augemented. For. ——. dooth as well augemente the
of Surde numbers.

the name, as — √ — dooth.

Scholar. It seemeth reasonable, for when I saie, √ 12. — — √ 7, the name is compound, an well as if I had saied, √ 12. — — √ 7, although the quanti-
tie bee not so greate. For — — dooth ever abate the
quattue of the number, though it do increase the name.

Master. Yet for a difference, the names that be
compounde with — — be called Bimiedialles; and those Bimiedialles
that be compounde with — — be named Residualles. Residualles.

And if the Bimiedialles have all their numbers and par-
tes of one denominations, then bee they called only
by their generall name Bimiedialles. But if their par-
tes be of 2 denominations, then are they named Bin-
omialles properly. Whereby, many use to call Binomialles Binomialles.

all compounde numbers that have — —. And so wil
I let the names passe.

Euclides definitions doe not very aptly agree to this
place, as at another time I will shewe you, and there-
fore I doe omitte them for this time.

But touching our principall intent, which is
to declare the practice whereof Binomialles, and Res-
idualles, there is little diffiultie, if you mark: well that
whiche is taught before. For as Binomialles and Res-
idualles, bee made of Surde, or els of rationalle numbers
with Surdes, so the worke of the compounde numbers
dependeth of the worke of the simple numbers, and is
all one with them. And concerning the signes — —
and. — —. here is no more to bee saied, then was
taughte in Cosike numbers compounde.

Scholar. Yet of every kynde, it maie please you to
set fo rthe some examples.

Master. I thinke that mete, without many wor-
des els. Not forgetting by the waie, that universalle
roots, are not accompted amonge these compounde
Surdes: but are referred to their peculiar treatise, as
roots of compounde Surdes.
The Arte
Of Numeration.
Numeration is moare plain, then that I neede to stande in declaring it, otherwaies than by examples:
As heri you see.

Examples as Binomialles.
6. —— √.8. That is 6 more the Square roote of 8.
√.20 ——.3. Is the Square roote of 20. moare.3.
√.30 —— √.9. Signieth the Cubike roote of 30. moare the zenzizenzike roote of 9.
And so of other.

Examples of Residualles.
24. —— √.96. That is 24. abating the roote of 96.
√.5208 —— √.35. The zenzizenzike roote of 5208.
laue the Square roote of 35. And so forthe.

Scholar. So I see any Surdes mate bee compounde with other: And any nöbers racionalle joined with the.

Of Addition.
Master. Addition is as paline. For as the partes bee, so shal the Addition bee, accordyng as you have learned before.

Examples of Binomialles.

\[
\begin{array}{l}
\sqrt{50} + 10 = \sqrt{15} \quad \sqrt{15} + \sqrt{135} = \sqrt{1264} + 8 \\
\sqrt{2} + 8 = \sqrt{18} + \sqrt{60} \quad 28 + \sqrt{316} = 36 + \sqrt{2844} \\
\sqrt{72} + 18 = \sqrt{133} \quad \sqrt{1264} + 8 \\
\end{array}
\]

\[
\begin{array}{l}
\sqrt{48} + \sqrt{5} = \sqrt{32} + \sqrt{10} \\
\sqrt{243} + \sqrt{45} = \sqrt{4} + \sqrt{19} \\
\sqrt{1875} + \sqrt{80} = \sqrt{108} + \sqrt{29} + \sqrt{760} \\
\end{array}
\]
Of Surde numbers.

Examples of Residualles.

\[ \sqrt{75} = 4 \quad 14 \quad \sqrt{3} \quad 25 \quad \sqrt{108} \]
\[ \sqrt{3} = 1 \quad 16 \quad \sqrt{27} \quad 44 \quad \sqrt{76} \]
\[ \sqrt{108} = 5 \quad 30 \quad \sqrt{12} \quad 174 \quad \sqrt{275} \]

\[ \sqrt{72} = \sqrt{96} \quad \sqrt{32} = \sqrt{5} \]
\[ \sqrt{9} = \sqrt{6} \quad \sqrt{32} = \sqrt{24} \]
\[ \sqrt{243} = \sqrt{162} \quad \sqrt{512} = \sqrt{29} + \sqrt{480} \]

Examples of Binomialles with Residualles.

\[ \sqrt{80} = 6 \quad 10 = \sqrt{20} \quad 561 = \sqrt{512} \]
\[ \sqrt{5} = 2 \quad 12 = \sqrt{5} \quad \sqrt{288} = 340 \]
\[ \sqrt{125} = 4 \quad 42 = \sqrt{5} \quad 901 = \sqrt{1568} \]

Scholar. I see that you make severalle Additions in all these numbers. For you adde still like numbers with their matches. So that there is nothing diverse from the woorkes of simple Surdes. Although in every thirde example, there appeare more difficultie, then there is in deed: When I consider the like transposition in Cosike numbers. For the woorke addeth like numbers together.

Of Subtraction.

Master. In Subtraction there is as litle diversite. As these examples will sufficiently declare: Where he set as trialles of the former Additions.

Q.q. Examples.
The Aree

Examples of Binomiales.
\[
\begin{array}{c|c}
\sqrt{.72} & 18 \\
\sqrt{.2} & 8 \\
\sqrt{.5} & 10 \\
\hline
36 & \sqrt{28.44} \\
\sqrt{1.264} & 8. \\
28 & \sqrt{3.16} \\
\hline
33 & \sqrt{1.35} \\
15 & \sqrt{1.5} \\
18 & \sqrt{.60} \\
\hline
\sqrt{1.08} & \sqrt{.29} & \sqrt{.76} \\
\hline
\sqrt{4} & \sqrt{.19} \\
\hline
\sqrt{32} & \sqrt{.10} \\
\end{array}
\]

Examples of Residualles.
\[
\begin{array}{c|c}
\sqrt{1.08} & 5 \\
\sqrt{3} & 1 \\
\sqrt{75} & 4 \\
\hline
174 & \sqrt{.275} \\
\sqrt{44} & 76 \\
250 & \sqrt{1.08} \\
\hline
30 & \sqrt{1.2} \\
14 & \sqrt{.3} \\
16 & \sqrt{.27} \\
\hline
\sqrt{243} & \sqrt{1.62} \\
\sqrt{9} & \sqrt{.6} \\
\sqrt{72} & \sqrt{.96} \\
\hline
\sqrt{512} & \sqrt{29} & \sqrt{480} \\
\sqrt{32} & \sqrt{5} \\
\hline
\sqrt{32} & \sqrt{24} \\
\end{array}
\]

Examples of bothe together.
\[
\begin{array}{c|c}
\sqrt{1.25} & 4 \\
\sqrt{5} & 2 \\
\sqrt{80} & 6 \\
\hline
901 & \sqrt{1568} \\
\sqrt{288} & 340 \\
561 & \sqrt{512} \\
\hline
42 \\
\end{array}
\]
of Surde nombers.

\[
\begin{array}{ccc}
42 + & \sqrt{5} & = \sqrt{112} = 11.48 \\
12 + & \sqrt{5} & = \sqrt{7} = 2.65 \\
30 + & \sqrt{2} & = \sqrt{63} = 7.94 \\
\hline
\end{array}
\]

\[
\begin{array}{ccc}
\sqrt{1080} + & \sqrt{80} & = \sqrt{5376} \\
\sqrt{40} + & \sqrt{24} & = \sqrt{56} \\
\hline
\end{array}
\]

Scholar. This is as easie as Addition, saue fo23. examples, which I understande not. Fo2 although I see the laste example, of eche of the sorte of nombers, to bee agreeable with the like examples in Addition, yet I can not so well perceiue, the order of their Subtraction, as I doe knowe the maner of their Addition. Fo2 by the arte of simple Surdes, I see that \(\sqrt{10} \) and \(\sqrt{19} \) doe make \(\sqrt{29} = \sqrt{760} \). But when \(\sqrt{29} = \sqrt{760} \) is set as a totall, and \(\sqrt{19} \) to be Subtracted out of it, how I shall woorke that, and leaue \(\sqrt{10} \), the remainder, I see not.

So in the residuall, I knowe how \(\sqrt{5} \) and \(\sqrt{24} \) doe make \(\sqrt{29} = \sqrt{480} \). But I knowe not how \(\sqrt{5} \) abated out of \(\sqrt{29} = \sqrt{480} \) doth make for the remainder \(\sqrt{24} \).

And the like doubt is in the thirde sorte of Surdes, which are mirte nombers. Fo2 where I see in Addition \(+\) \(\sqrt{24} \) added with \(-\) \(\sqrt{56} \). And the totalle to bee \(-\) \(\sqrt{80} \) \(-\) \(\sqrt{5376} \). I knowe the reason of the woorke, fo2 the signes: \(-\) \(+\) \(-\). By that I learned in Cosike nombers: And the reaftes is manifeste by Addition of simple Surdes. Fo2 it is wrought by abating \(\sqrt{24} \) out of \(\sqrt{56} \). But then in Subtraction, how \(-\) \(\sqrt{24} \) being Subtracted from \(-\) \(\sqrt{80} \) \(-\) \(\sqrt{5376} \) shall leaue \(-\) \(\sqrt{56} \) I can not judge. And yet by the signes I geffe (as I learned in Cosike nombers) that it is done by Addition, because the signes doe disagree.

The Arte

Mater. In that you remember the former rules, to conferre them aptly with these later workes, I can praise you well. But in that you cannot understande the reason of that, which was not yet taungte you, I can not greatly blame you. Although I can not praise you, so that you thinke your self to be cunningger then you are. For in those Additions, that you thinke your self to be experte enough, I dare saye, that you bee discouerd, if you take them to bee numbers of any soche, as hitherto hath been taungte vnto you.

Scholar. I take them for compounde Surdes.

Mater. Thei are not so: Nother is their wooyke agreeable, with the wooyke of compounde Surdes. But thei are the rootes of compounde Surdes: And therefore are called universalle rootes of Surdes. And accordingly to their proper nature, thei ought to bee called rootes of Surdes, and not Surde rootes. As I will tell you anon. When I will also discusse your doubte.

But before I speake any moare of them, I will cande the wooykcs of these compounde Surdes: whereof 2. kindes yet remaine behinde.

Of Multiplication.

Multiplication of compounde Surdes, is as easie as can bee. And differeth in nothynge, fro the worke of simple Surdes. Onely this must you marke, as reason would, that you muste multiplye euery parte of the one nober, by euery parte of the other nober: as you remember the worke of compounde Coficke numbers.

Scholar. I praze you giue me some cromyles.

Mater. That shal you haue. And that same suffece so; this wooyke. Marke them well therefore.

Cromyles
of Surde nombers.

Examples of Binomialles.

\[ \sqrt{23} = \sqrt[.15]. \]
\[ \sqrt{6} = \sqrt[.8]. \]
\[ \sqrt{138} = \sqrt[.120]. \]
\[ \sqrt{138} = \sqrt[.4232] + \sqrt[.540] + \sqrt[.120]. \]
\[ \sqrt{.120} = \sqrt[.12]. \]
\[ \sqrt{.12} = \sqrt[.7]. \]
\[ \sqrt{1440} = \sqrt[.84]. \]
\[ \sqrt{1440} = \sqrt[.840] + \sqrt[.12]. \]
\[ \sqrt{12} = \sqrt[.1440] + \sqrt[.840] + \sqrt[.84]. \]

Examples of Residualles.

\[ \sqrt{5} = \sqrt[.10]. \]
\[ \sqrt{5} = \sqrt[.10]. \]
\[ \sqrt{25} = \sqrt[.10]. \]
\[ \sqrt{25} = \sqrt[.250] + \sqrt[.250]. \]
\[ \sqrt{35} = \sqrt[.1000]. \]
\[ \sqrt{.24} = \sqrt[.20]. \]
\[ \sqrt{.30} = \sqrt[.24]. \]
\[ \sqrt{.720} = \sqrt[.480]. \]
\[ \sqrt{.720} = \sqrt[.480] + \sqrt[.600]. \]
\[ \sqrt{.720} = \sqrt[.480] + \sqrt[.600]. \]

Examples of bothe together.

\[ \sqrt{32} = \sqrt[.14]. \]
\[ \sqrt[.124] = \sqrt[.6]. \]
\[ \sqrt[.126976] = \sqrt[.1736]. \]
\[ \sqrt[.126976] = \sqrt[.1736] + 192 = \sqrt[.504]. \]
\[ \sqrt[.126976] = \sqrt[.1736] + 192 = \sqrt[.504]. \]
\[ \text{Req'd.} \quad \sqrt[.52]. \]
The Arte

\[ \sqrt{15028} = 38.9 \]

Scholar. Multiplication, as I see, is the easiest workke of all the other. So that I doo marke the reduction, in gathering the totall: Whiche is easie enough to understand, by that I have learned in Cofike numbers. And Division be no harder, it maye tone be learned.

**Of Division.**

Master.

Division by one simple number, is no more difficulte: as these examples doe declare. Where the divisor is a number compound. 

\[ \sqrt{26} = 15 \text{ divided by } 5 \text{ doeth make. } \sqrt{1} \]

Again. \[ \sqrt{56} = \sqrt{2} 4 \text{ divided by } \sqrt{6} \text{ doeth yeilde. } \sqrt{9} \]

And so \[ \sqrt{75} = \sqrt{2} 4 \text{ divided by } \sqrt{3} \text{ dooth byng for the } 5. \]

Like waies. \[ \sqrt{32} = \sqrt{1} 8 \text{ byng parted by } \sqrt{5} \text{ doeth make the quotiente. } 14. \]

Scholar. I see it so. For at the firste it is. \[ \sqrt{6} 4. \]

Master. So may you woorkke all like divisions. But when the divisor is a compound number, then must you use another mean: that is to reduce that compound number, to a simple number: Whiche thing you maye easely doe, by multiplying any Binomiale, by his Residuale, or contrary waies, the Residuale by his Binomiale.
of Surde nombers.

As $\sqrt{6} + \sqrt{\frac{10}{6}}$ multiplied by $\sqrt{6} = \sqrt{\frac{6}{6}}$ doeth make $\sqrt{10}$.

And so $\sqrt{\frac{8}{6}}$ multiplied by $\sqrt{\frac{5}{8}} = \sqrt{\frac{8 	imes 5}{6 	imes 8}}$ doeth yeild $\frac{5}{8}$. That is $\frac{5}{8}$.

Scholar. I perceive a brief ware in this multiplication: For if I neede not in the ifst example, to multiply $\sqrt{6}$ by $\sqrt{\frac{10}{6}}$, it would amounte to nothing. In so moche as at one multiplication, it would bee $\sqrt{\frac{6}{6}}$, and at another, $\sqrt{\frac{8}{6}}$. And so the one would abate the other, and leave nothing for them bothe.

Mater. That is well marked. And it is so generally. Wherefore (as you see) the divisor, by this meanes, maye lightly be tourned into a simple nomber, or a plaine absolute nomber.

And now to make the diuidede, in the same proporction, as this navel diuisor, that it was unto the old diuisor, you shall multiply it by the same nomber, by whiche the diuisor was multiplied. For if any nombers bee multiplied, by one common nomber, their navel totalles kepe the same proportion, that was betweene the firste nombers.

Scholar. That must needs be so. For as $\frac{3}{6}$ is sesquialtera unto $\frac{2}{3}$, so if you multiply them by $\frac{5}{3}$, they will make $\frac{15}{6}$ and $\frac{10}{3}$. Whiche bee in sesquialtera proportion and likewaies will their proportion remain, by what so euer nomber they be multiplied. Wherefore it must needs be reasonable, that if the diuidende and the diuisor, be multiplied by any one nomber, simple or compounde, they shall kepe the same proportion, that they had before.

Mater. For more certain understanding of this rule, take these examples. The ifst is, where $\sqrt{6} + \sqrt{\frac{10}{6}}$. is sette to bee diuided by $\sqrt{\frac{6}{6}} + \sqrt{\frac{8}{6}}$.

Here ifst I multyplace the diuisor by his contrarie, that is his Binomi:

\[
\begin{array}{c|c|c}
\sqrt{\frac{6}{6}} & + & \sqrt{\frac{8}{6}} \\
\sqrt{6} & + & \sqrt{\frac{5}{6}} \\
\hline
6 & + & 3 \\
\hline
\end{array}
\]

That is $\frac{5}{6}$.
The Arte

alle. $\sqrt[6]{3}$ And there riseth $6$, that is, $3$ whiche I shall kepe for the newe diuifor

Then doe I multiplye the dividdede $\sqrt[6]{6}$ by the same Residua. $\sqrt[5]{4}$

$$
\begin{array}{c}
\sqrt{68} \\
\sqrt{6} \\
\hline
\sqrt{324} \\
\sqrt{204} \\
\sqrt{162} \\
\sqrt{162} \\
\end{array}
$$

And therfore as here in woork is expresst.

$$
\begin{array}{c}
\sqrt{408} \\
\sqrt{324} \quad \sqrt{204} \quad \sqrt{162} \\
\sqrt{9} \\
\end{array}
$$

whiche number shall be taken for the newe dividide: and must be dividide by $3$, that is the newe divisor. In whose these I let $\sqrt{9}$ mroe redinelle in woork.

Therefoe I set the dounie in order, as here foloweth.

$$
\begin{array}{c}
\sqrt{408} \\
\sqrt{324} \quad \sqrt{204} \quad \sqrt{162} \quad \sqrt{45} \quad 6 \quad \sqrt{22} \quad \sqrt{18} \\
\sqrt{9} \\
\end{array}
$$

And then doe I seke how often $\sqrt[9]{9}$ male bee founde in $\sqrt{408}$ whiche male bee $\frac{45}{3}$ of tymes. Therfore I set $\sqrt{45}$ in the quotiente. And then doe I reiterate the divisors, and sette it under $\sqrt{324}$, where I finde it $36$ tymes: and therfore set I $6$ for it, because the quotiente els would bee $\sqrt{36}$ whiche is rusticly $6$.

Thirdly, I remowe the divisor under $\sqrt{204}$ where it male bee founde $\frac{22}{3}$ tymes. For whiche I sette $\sqrt{22}$ in the quotiente. And then set I the divisor last of all under $162$ where it is founde $18$ tymes: and for that cause I set $\sqrt{18}$ in the quotiente: And so is the whole quotiente $\sqrt{45} \quad 6 \quad \sqrt{22} \quad \sqrt{18}$

Scholar. This division is straunge to credite, although it be not dificulte to woork.

Master. If you doubt of it, you maie be the accustomed triall by the contrary kinde.

Scholar.
of Surde nombers.

Scholar. So must it followe, that if I doe multiply this quotiente by the firste diuision, the firste dividende will resulte thereof.

And for the prooffe of that, I doe multiply,

\[ \sqrt{45\frac{1}{3}} - \sqrt{6} - \sqrt{22\frac{1}{3}} - \sqrt{18}. \]

by \[ \sqrt{6} - \sqrt{3}. \]

But for the more ease, I doe turne all the mirte nombers into onely fractions. And then doe I multiply them orderly.

\[ \sqrt{\frac{135}{3}} - 6 - \sqrt{\frac{68}{3}} - \sqrt{18}. \]

\[ \sqrt{6} - \sqrt{3}. \]

\[ \sqrt{81\frac{1}{3}} + \sqrt{216} - \sqrt{\frac{408}{3}} - \sqrt{108}. \]

\[ \sqrt{4\frac{8}{3}} + \sqrt{108} - \sqrt{\frac{204}{3}} - \sqrt{54}. \]

\[ \sqrt{272} - \sqrt{216} - \sqrt{136} - \sqrt{108}. \]

\[ \sqrt{68} - \sqrt{54} - \sqrt{136} - \sqrt{108}. \]

\[ \sqrt{68} - \sqrt{54}. \]

First I multiply \( \sqrt{\frac{135}{3}} \) by \( \sqrt{6} \), and there commeth \( \sqrt{\frac{81}{3}} \) that is \( \sqrt{272} \). Again I doe multiply \( \sqrt{6} \) by \( \sqrt{6} \) and it maketh \( \sqrt{216} \). Then I multiply \( \sqrt{\frac{408}{3}} \) by \( \sqrt{6} \), it giueth \( \sqrt{\frac{204}{3}} \), which is \( \sqrt{136} \). Fourtieth \( \sqrt{18} \) multiplied by \( \sqrt{6} \), dooth make \( \sqrt{108} \). All which I set doone with their convenient signs.

After that I multiply \( \sqrt{\frac{135}{3}} \) by \( \sqrt{3} \), and it yieldeth \( \sqrt{\frac{428}{3}} \) that is \( \sqrt{136} \), which I sette doone with his signe — — . Then \( \sqrt{36} \) by \( \sqrt{3} \), maketh \( \sqrt{108} \). Thirdly \( \sqrt{\frac{68}{3}} \) by \( \sqrt{3} \), dooth giue \( \sqrt{68} \), and last of all, \( \sqrt{18} \) multiplied by \( \sqrt{3} \), byngeth for the \( \sqrt{54} \).

When all these be placed conveniently, I doe consider that — — \( \sqrt{136} \) and — — \( \sqrt{136} \), maie bec bothe cancelled, because the one dooth abate the other. And likewaies, — — \( \sqrt{108} \), and — — \( \sqrt{108} \). Eche abate other: so that thei must bothe be reteched.

Then I see, that \( \sqrt{68} \) byng abated out of \( \sqrt{272} \) there will remain, \( \sqrt{68} \). And in like, \( \sqrt{54} \) byng abated out of \( \sqrt{216} \), dooth leaue \( \sqrt{54} \). So that the whole multiplicatiō both make usuflly \( \sqrt{68} - \sqrt{54} \)

\[ \sqrt{54} \]

whiche
The Arte

which is the first dividende. And so is that division approved good.

Master. Yet for you exercise, you shall have some examples more of division.

\[ \sqrt{456} - \sqrt{72} \text{ is lette to bee divided by} \]

\[ \sqrt{18} - \sqrt{6} \]

Scholar. That diuisor must I \[ \sqrt{18} - \sqrt{6} \]
multiply by his contrarie, which is the Residuall. \[ \sqrt{18} - \sqrt{6} \]
so, as you make some perceive, \[ \frac{18}{6} = 3 \]
there will rise. \[ 18 - 6 \] that is 12. which must be kepted for the newe diuisor.

Then shall I multiplie the former dividende, that is \[ \sqrt{456} - \sqrt{72} \] by the same residuall \[ \sqrt{18} - \sqrt{6} \]

\[ \begin{array}{c}
\sqrt{456} - \sqrt{72} \\
\sqrt{18} - \sqrt{6}
\end{array} \]

\[ \sqrt{8208} - \sqrt{1296} \]
\[ \sqrt{432} - \sqrt{2736} \]

\[ \sqrt{8208} - \sqrt{432} - \sqrt{2736} - \sqrt{1296} \]

And there will rise of that multiplication, as here by example appereth \[ \sqrt{8208} - \sqrt{432} - \sqrt{2736} - \sqrt{1296} \] which nober I shall divide by 12. that was founde for the newe diuisor. And then will the
quotiente bee, \[ \sqrt{57} - \sqrt{3} - \sqrt{19} - \sqrt{9} \]
As here in woozke doth appeare.

\[ \sqrt{8208} - \sqrt{432} - \sqrt{2736} - \sqrt{1296} \]
\[ \sqrt{144} - \sqrt{144} - \sqrt{144} - \sqrt{144} \]

Where I haue set, \[ \sqrt{144} \] for 12. scyng that be all one; but that, \[ \sqrt{144} \] is more apte for this woozke.
And I haue repeated it as often tymes, as the diuisor should be removed.

The proofe. But now to trie this woozke, whether it bee well wroughte, I shall multiplie this quotiente by the first diuisor, I then ought the first dividende to amounte.

As
Of Surde numbers.

As here in example, you see wrought.

\[ \sqrt{57} \quad \sqrt{3} \quad \sqrt{19} \quad \sqrt{9} \]

\[ \sqrt{18} \quad \sqrt{6} \]

\[ \sqrt{1026} \quad \sqrt{54} \quad \sqrt{342} \quad \sqrt{162} \]

\[ \sqrt{342} \quad \sqrt{18} \quad \sqrt{114} \quad \sqrt{54} \]

\[ \sqrt{1026} \quad \sqrt{18} \quad \sqrt{114} \quad \sqrt{162} \]

Where \[ \sqrt{54} \] doth cancel \[ \sqrt{54} \] and is cancelled by it.

So \[ \sqrt{342} \] and \[ \sqrt{342} \] exclude one an other, and therefore must be both rejected. And then remaineth onely,

\[ \sqrt{1026} \quad \sqrt{18} \quad \sqrt{114} \quad \sqrt{162} \]

Whiche numbers I doe well examine: and finde that \[ \sqrt{114} \] byng abated out of \[ \sqrt{1026} \] there will remaine \[ \sqrt{456} \]. Againse if \[ \sqrt{18} \] be subtracted out of \[ \sqrt{162} \] there will reste \[ \sqrt{72} \]. And so is that whole multiplicatio onely \[ \sqrt{456} \quad \sqrt{72} \] agreeable to the firste divine. Whereby it is manifestlye, that the former division was good.

Master. How can you wooze this example?

Where \[ 24 \] is set to be divied by \[ 3 \] \[ \quad \sqrt{8} \]

Schollar. I must still obserue the generall rule. And multiply bothe those numbers, by the contrarie of the divisor, that is, by the residuall \[ 3 \quad \sqrt{8} \] And of the firste multiplication of it, with the dividend, \[ 24 \] there riseth \[ 72 \quad \sqrt{4608} \] Of the seconde multiplica- tion, where the Binomiale is multiplied by the Residuall, that is his contrarie, the totalle will be \[ 9 \quad \sqrt{8} \]. That is but \[ 1 \]. That is \[ 1 \]. And therefore sayng \[ 1 \] doeth not other multipliue noz divide, the former number.

That is \[ 72 \quad \sqrt{4608} \] is the quotiente, when \[ 24 \] is divided by \[ 3 \] \[ \quad \sqrt{8} \]

Mr. s. For
The Art.

The profe. If I prove wherof I multiply 72 —— v. 4608.

that is the quointe by 3 —— v. 8. And there riseth

216 —— v. 41472 —— v. 41472 —— v. 36864.

Wherof 2. numbers differing by 3 —— v. 8.
musie bothe bee rejected, as numbers superflouose.

Then. 36864 is a square number, and hath. 192 for his root. Wherof the whole number is,

216 —— 192 that is (as it is manifest enough) 24. And so is the whole woozie proved good.

The fourthe

example.

Master. You shall have one example more, and then I will make an ende of division.

When v. 6570. —— v. 254. is propounded to be divided by v. 54 —— v. 6. I would knowe the quointe.

Scholar. I see the newe diuisor will be, 54 —— 6.

that is 48.

And then so to finde a diuindende conveniante, I

v. 6570. —— v. 254.

v. 54 —— v. 6.

v. 354780 —— v. 13716.

v. 39420 —— v. 1524.

v. 13716 —— v. 39420 —— v. 1524.

That diuindende must be divided by 48, or more app
tly by v. 2304. And the quointe will bee.

v. 15 3 10 3 3 —— v. 15 3 10 3 3 —— v. 17 63 —— v. 17 63 —— v. 17 63 —— v. 17 63.

As here appeareth in woozie.

v. 354780 —— v. 13716 —— v. 39420 + v. 1524 (v. 15 3 10 3 3 —— v. 15 3 10 3 3 —— v. 17 63 —— v. 17 63 —— v. 17 63.

v. 2304 + v. 2304 + v. 2304 + v. 2304.

The profe. And that this woozie is good, I will prove it by multiplication.
of Surde nombers.

multiplication. As the example followynge dooth declare. Where by the firste multiplication there come meth. 8. nombers, that is. 4. with. ——. and. 4. with. ——.

\[
\sqrt{2956} \div 192 \quad \sqrt{1144} \div 192 \quad \sqrt{328} \div 192 \quad \sqrt{127} \div 192 \\
\sqrt{54} \div 6
\]

\[
\sqrt{1596510} \div 192 \quad \sqrt{61722} \div 192 \quad \sqrt{17759} \div 192 \quad \sqrt{192} \div 192 \\
\sqrt{1596510} \div 192 \quad \sqrt{17759} \div 192 \quad \sqrt{192} \div 192 \quad \sqrt{762} \div 192 \\
\sqrt{6570} \div 762 \\
\sqrt{254}
\]

And because the firste nober with ——, is equalle to the thirde with ——, thesone the bothe must be retracted. Again in as moche as the seconde nober with ——, is equalle to the fourthe number with ——, the bothe shall bee cancelled. And then remaineth. 2. nombers with ——, and other. 2. with ——.

So if you abate the thirde —— out of the firste ——, the quotiente will be. \(\sqrt{6570}\).

Likewise if you abate the fourthe —— out of the seconde ——, the quotiente will yeild. \(\sqrt{254}\).

And the bothe will make the firste dividende. \(\sqrt{6570}\). Wherby the former division is approv'd good.

Parker. This shall suffice for division.

Of extraction of rootes.

The nerte wozke is extraction of rootes: which you make very easilie wozke, by puttynge the signe of the roote, that you desire, before the whole number. As if you would have the square roote of \(\sqrt{10}\)

\[
\sqrt{5}, \text{this is it} \quad \sqrt{10} \quad \sqrt{5}. \text{The Cubike roote of the same nober is}, \quad \sqrt{\sqrt{10}} \quad \sqrt{5}. \text{And the senzizenzikike roote of it is} \quad \sqrt{\sqrt{\sqrt{10}}} \quad \sqrt{\sqrt{5}}.
\]

But if you will have the square roote of. \(\sqrt{10} \quad \sqrt{\sqrt{5}}

\text{Mr. Mr.}
The Arte

it is \( \sqrt{16} - 4 = \sqrt{5} \). And his cubike roote is \( \sqrt[3]{10} = \sqrt[6]{2} \). Like-waies his zen-zen-zenike roote is \( \sqrt[6]{2} \).

So of \( \sqrt[6]{2} \) the square roote is \( \sqrt[6]{2} \cdot \sqrt[6]{2} = \sqrt[6]{18} \). The cubike roote is \( \sqrt[6]{18} \cdot \sqrt[6]{2} = \sqrt[6]{36} \). And the zen-zen-zenike roote is \( \sqrt[6]{36} \).

Scholar. Hereby I perceive that the later parte of the composition, is not varied at all, but onely the friste parte taketh buto to it the signe of the roote. And that signe is referred to the whole compounde number.

Master. These rootes therefore bee called vniversalle rootes, because they are the rootes, not of the seueralle partes of the compounde number, but of the whole compounde number. And that is the difference, twixt the common Surde numbers, and vniversalle rootes. For if \( \sqrt{24} = \sqrt{144} \) be sette fo\( 2 \) a common Surde number, then doeth it betoken, that I must take 2. rootes, that is \( \sqrt{24} \) and \( \sqrt{144} \), andJoyne them together. But if it stannde fo\( 2 \) an vniversalle roote, it representeth the roote of this whole number. \( 24 + \sqrt{144} \) which is 6. so the whole square is 36.

Scholar. I perceive it well. For \( \sqrt{144} \) beynge 12, that 12. with 24 dooth make 36. And therefore must the vniversalle roote of \( 24 - \sqrt{144} \) bee 6. And fo \( 24 + \sqrt{144} \) is 36.

But if \( 24 + \sqrt{144} \) doe stannde fo\( 2 \) a common Surde number compounde: then is it made of 2. rootes, that is \( \sqrt{24} \) whiche is almoste 5. and \( \sqrt{144} \) beynge 12. And so the whole compounde roote, in that sorte is almoste 17. And is nigher 3. tymes so moche as the same number, byynge an vniversalle roote.

Master. Because you maye perceive it the better, I will put an example in square numbers, made like Surdes. As this. \( \sqrt{81} - 1 = \sqrt{36} + 1 \) if it be an vniversalle roote, then it is equalle to 16. For I must take first the roote of the laste number, which is 19. And add to it with
of Surde nombers.

with. 81. Wherby there amounteth. 10. Whose roote is. 10. But if it stand after the common loyte of Surde nombers, it betokeneth the roote of. 81. and the roote of. 361. (that is. 9. and. 19) to bee added together. And so thei make. 28. whiche is farre aboue. 10.

But farther now, if it stande for a common Surde number: And I would have the Square roote of it, then is that. \( \sqrt{81} = 9 \) and \( \sqrt{361} = 19 \). And betokeneth the Square roote of the Square roote of. 81. and the Square roote of. 361. added together, that is the Square roote of. 28. But moister generally and moister aptly, it betokeneth the roote of the universalle roote of. 81. and. 361.

Scholar. Now I perceive that in Addition, and Subtraction of Surdes, the last nombers that did resulde of that woorkes, were universalle rootes.

Master. You sawe truth. But harke what meaneth that hastie knockynge at the doore?

Scholar. It is a messenger.

Master. What is the message? tel me in mine care. Psea sir is that the mater? Then is there noe remec-dic, but that I must neglect all studies, and teaching, so to withstande those daungers. Mye fortune is not soo good, to have quicke tymen to teache.

Scholar. But my fortune and mye fellowes, is moche worse, that your unquietnes, so hindereth our knowledge. I praye God amende it.

Master. I am insoxed to make an cande of this mater: But yet will I promise you, that whiche you shall chalenge of me, whi you see me at better latere: That I will teache you the whole arte of universalle rootes: And the extraction of rootes in all Square Surdes: With the demonstration of them, and all the former woorkes.

If I mighte have beene quietly permitted, to rese but a little while longer, I had determined not to have ceased, til I had ended all these thinges at large: But now
The Arte

now farewell. And apply your studie diligently in this that you have learned. And if I make yette any quietnesse reasonable, I will not forget to performe my promise with an augmentation.

Scholar. My harte is so oppresssed with pambilnes, by this sodaine unquietnesse, that I can not expressse my grief. But I will praise, with all them that love honeste knowledge, that God of his mercie, will some ende your troubles, and graunte you soche rest, as your travell dooth merite. And al that love learning: sake thereto. Amen.

Master, Amen, and Amen.

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